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EXERCISES
IN
ELECTRICAL AND MAGNETIC MEASUREMENT

R. E. DAY.

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ELECTRICAL AND MAGNETIC MEASUREMENT

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EXERCISES

IN

ELECTRICAL AND MAGNETIC MEASUREMENT

BY

R. E. DAY, M.A.

LATE EVENING LECTURER IN EXPERIMENTAL PHYSICS
AT KING'S COLLEGE

NEW EDITION

LONDON
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PREFACE.

IN the composition of this little book I have endeavoured to arrange in a systematic form a collection of Exercises in Electricity and Magnetism for the use of students in general, and more especially for those who are not able to command the services of a private tutor.

My object has been to lay before the student, under the form of problems, numerical illustrations of the main facts of Electricity and Magnetism, with especial reference to the modern doctrine of Energetics.

It is now universally admitted that numerical exercises are necessary in the study of the experimental sciences, both as giving practice in the application of the various theories and as affording tests of ability to comprehend as well as to apply that which has been learned. A thorough grasp and realisation of the meaning of a formula in Physics can scarcely be acquired from lectures, or from seeing it in the form of algebraical symbols in a text-book, whereas the working out by the student himself of its application to a single concrete case is often sufficient to make him feel confidence in using it, and to give precision to ideas which would otherwise remain floating in his mind in a vague and comparatively useless form.

In support of this view, I need only quote the words used by Sir W. Thomson at the commencement of a lecture delivered on May 3, 1883, at the Institute of Civil Engineers. 'In physical science a first essential step in the direction of learning any subject is to find principles of numerical reckoning, and methods for practically measuring some quality connected with it. I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of *science*, whatever the matter may be.'

I am indebted to Mr. C. D. Webb, A.K.C., for much assistance in checking the numerical results, but, as we cannot hope to have been entirely successful, I shall feel obliged to any student who will inform me of any inaccuracies which may have escaped our notice.

The present edition differs so much from the former one that the book is practically a new one. The alterations are such as I have found to be desirable in order to make the book useful to my own students, many of whom are themselves science teachers, or are professionally engaged in Electrical Engineering.

R. E. DAY.

UNITS.

THE quantitative relations between the various forms of energy necessitate the reduction of all kinds of physical quantities to one common scale of comparison. To facilitate this process several systems of so-called absolute measurement have been suggested by scientific men, but the diversity which existed until recently in the selection of the fundamental units of length, mass, and time, has greatly interfered with their practical usefulness.

Until there is a general agreement as to the selection of the fundamental units, it is necessary for every person who wishes to specify a magnitude in what is called *absolute* measure to mention the three fundamental units of length, mass, and time, which he has chosen as the basis of his system. This necessity is obviated if one definite selection is made once for all, and is accepted by the general consent of scientific men.

The result of the labours of a succession of committees has been the general adoption of the centimetre, the gramme, and the second as the three fundamental units; and the units of electrical and magnetic magnitude hence derived are distinguished from *absolute* units otherwise derived by the letters C.G.S. prefixed.

The word *absolute* does not here mean that the measurements or units are absolutely correct, but only that the measurement, instead of being a simple comparison with an arbitrary quantity of the same kind as that measured, and therefore a *relative* one, is made by reference to certain fundamental units of another kind, treated as postulates.

The absolute magnitude in many cases necessitates the use of exceedingly large or exceedingly small numbers, and the best way of writing these numbers is to express them as the product of two factors one of which is a power of 10, and to effect the resolution in such a way that the exponent of the power of 10 shall be the characteristic of the logarithm of the number. In practice, decimal multiples and submultiples of these units are generally adopted.

Fundamental Units.

Length = one centimetre = ·3937 inch.
Mass = one gramme = 15·432 grains.
Time = one second.

The C.G.S. unit of force is called the *dyne*. It is the force which, acting on a gramme for a second, generates a velocity of one centimetre per second.

Force is said to be expressed in *gravitation-measure* when it is expressed as equal to the weight of given mass. In order that such a specification should be exact, the value of g at the place of observation must also be given.

A force of one dyne is very nearly equal to the weight of 1·02 milligramme anywhere on the earth's surface.

The C.G.S. unit of work is called the *erg*. It is the amount of work done by one dyne working through a distance of one centimetre.

One gramme-centimetre is equal to g ergs. In London g is about equal to 981 C.G.S. units of acceleration. One foot-pound $= 13825 \times g$ ergs.

The C.G.S. unit rate of working, or *unit of power*, is one erg per second. Taking Watt's horse power at 550 foot-pounds per second, we should have one 'horse power' equal to $7{\cdot}46 \times 10^9$ ergs per second. The French *force de cheval* being defined as 75 kilogrammetres per second, would make one force de cheval = $7{\cdot}36 \times 10^9$ ergs per second, if $g = 981$.

For multiplication or division by a million the prefixes *mega*, or *megal*, and *micro* are employed. Thus a *megalerg* is a million ergs, and a microdyne is the millionth part of a dyne.

Electrostatic Units.

The C.G.S. electrostatic unit quantity of electricity is that quantity which at a distance of one centimetre repels a similar and equal quantity with a force of one dyne.

The C.G.S. unit difference of potential is that difference through which the unit quantity must be raised that the work done may be one erg.

The C.G.S. unit of capacity, being that of a conductor which requires a charge of one unit of electricity to raise it to unit potential, is equal to the capacity of an isolated sphere of one centimetre radius.

Magnetic Units.

In the C.G.S. system the unit magnetic pole, or the pole of unit strength, is that which at a distance of one centimetre repels an equal and similar pole with a force of one dyne.

The intensity of a magnetic field is equal to the force which a unit pole experiences when placed in it. The field of unit intensity is that which acts with a force of one dyne on the unit magnetic pole.

The moment of a magnet is the product of the strength of either of its poles by the distance between them.

Electro-magnetic Units.

The C.G.S. electro-magnetic unit of current is a current of such a strength that if it were made to flow round a circuit in the form of a circle of one centimetre radius, the current in an arc of one centimetre would exert a force of one dyne on a unit magnetic pole at the centre.

The C.G.S. electro-magnetic unit of quantity is the quantity which is conveyed by the unit current in one second. The electro-magnetic unit of quantity is equal to about 3×10^{10} times the electrostatic unit.

The unit difference of potential, or the unit of electromotive force, exists between two points when one erg of work has to be done in order to urge a unit of positive electricity from the point at the lower to that which is at the higher potential.

The unit of resistance is possessed by a conductor if a current of unit strength flows through it when its ends are kept at unit difference of potential.

UNITS EMPLOYED BY PRACTICAL ELECTRICIANS.

(1) The unit of resistance employed by practical electricians is the ***ohm.*** The legal ohm is equal to the resistance of a column of pure mercury one square millimetre in section and 106 centimetres long at the temperature of melting ice.

One ohm = 10^9 absolute electro-magnetic C.G.S. units of resistance.

(2) The practical unit of electromotive force is called the *volt.*

One volt = 10^8 C.G.S. unit of potential.

(3) The practical unit of current is called the ***ampère.*** It is the current due to an electromotive force of one volt working through a resistance of one ohm.

One ampère = 10^{-1} C.G.S. unit of current.

(4) The practical unit of quantity is called the *coulomb.* It is equal to $\frac{1}{10}$th of the C.G.S. electromagnetic unit of quantity.

(5) The capacity of a condenser which holds one coulomb when charged to a potential of one volt is called a *farad.*

One farad = 10^{-9} C.G.S. unit of capacity.

The farad being in general too large a unit for practical purposes, its millionth part, called a *microfarad*, is generally employed as the unit. The capacity of submarine cables is generally about $\frac{1}{3}$rd of a microfarad per knot.

(6) The practical electrical unit of power is called the *watt*. It is the power conveyed by a current of one ampère through the difference of potential of a volt.

$$\text{One watt} = 10^7 \text{ ergs.}$$

(7) The unit of heat based on the electromagnetic system is called the *joule*. It is the heat generated in one second by the current of one ampère flowing through a resistance of an ohm.

One joule = 10^7 C.G.S. units, and, assuming Joule's equivalent as $4{\cdot}2 \times 10^7$, it is the heat necessary to raise 0·238 gramme of water 1° C.

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EXERCISES

IN

ELECTRICAL MEASUREMENT.

ELECTROSTATICS.

(1.) Two small insulated metal spheres were charged, one with 12 units of positive and the other with 12 units of negative electricity, and were placed, first at a distance of 12, and then at 18 centimetres from each other. What was the attractive force between them at each of these distances?

In the C.G.S. system the electrostatic unit of electricity is that quantity which would attract or repel an equal quantity at the distance of one centimetre in air with a force of one dyne.

The expression for the mutual force between two quantities of electricity represented by q_1 and q_2 at the mutual distance d is

$$\text{F}=\frac{q_1\times q_2}{d^2},$$

and since the electricities are of opposite kinds the forces in the present case are attractive, and are given by

$$\text{F}_1=\frac{12\times 12}{(12)^2}=1 \text{ dyne}; \quad \text{F}_2=\frac{12\times 12}{(18)^2}=\tfrac{4}{9} \text{ dyne.}$$

(2.) If the same two spheres were charged, one with 8 units of +E and the other with 4 units of −E, what would be the attractive force between them at a distance of 15, and also at 10 centimetres?

Answer. 0·14$\dot{2}$ dyne and 0·32 dyne.

(3.) Two small insulated spheres are each charged with 5 units of +E. What must be their distance apart so that their mutual repulsion may be equal to one dyne?

Let x = required distance in centimetres; then by the conditions of the question we have

$$F = \frac{5 \times 5}{x^2} = 1 \text{ ; whence } x = 5 \text{ centimetres.}$$

(4.) Had the repulsive force between the two spheres been 0·25 dyne, what would have been their distance apart? *Answer.* 10 centimetres.

(5.) Two small insulated metal balls were charged with 20 units of +E and 15 units of −E respectively. They were then made to touch one another, and afterwards placed at a distance of 2 centimetres apart. With what force would they repel each other?

During contact the 15 units of −E would neutralise 15 units of +E, and the charge on each ball would then be 2·5 units of +E. Their mutual repulsive force is therefore

$$F = \left(\frac{2{\cdot}5}{2}\right)^2 = 1{\cdot}56 \text{ dynes, nearly.}$$

(6.) If the initial charges on the balls were + 20 and − 5 units and their distance apart, after contact, 3 centimetres, what would be the mutual repulsive force?

Answer. 6·25 dynes.

(7.) Two small spheres were charged with quantities of electricity in the ratio of 3 to 7, and when placed at a considerable distance apart were found to repel each other with a certain force; they were then brought in contact, and afterwards separated to three times their former distance.

Compare the repulsive forces between them before and after contact. *Answer.* 189 : 25.

(8.) The two spheres were next charged, one with 5 units of positive and the other with 3 units of negative electricity, and the force between them at a certain distance was measured. They were then brought in contact and replaced in their original positions. Compare the mutual forces between them in these cases. *Answer.* As 15 : 1.

(9.) Each of these spheres was then charged with positive electricity of a certain intensity, and they were found to repel one another with a force of 1 dyne. Each of them was then charged with positive electricity of double the former intensity. What was their mutual repulsive force? *Answer.* 4 dynes.

(10.) A plain metallic surface whose area is 48 square centimetres is charged with +E until the average electrical density is 3; and another plane metallic surface of 8 square centimetres area is charged with −E, so that its average electrical density is 4. Suppose the two surfaces be now connected, what will be the average electrical density on each?

Electrical density is a term used to denote the quantity of electricity per square centimetre on the surface of a charged body.

The charge of +E on first plate $= 48 \times 3 = 144$ units.
" " −E " second " $= 8 \times 4 = 32$ "
∴ Quantity of +E remaining after contact $= 112$ units.
∴ Electrical density on each after contact

$$= \frac{\text{charge}}{\text{area}} = \frac{112}{56} = 2.$$

(11.) If the larger surface had been charged with −E to an average electrical density of 5, and the smaller surface with +E to an average density of 4, what would have been the quality of the electricity and its density after contact?

Answer. Quality, negative. Density, 3·7 nearly.

(12.) Two plane conducting surfaces whose areas were respectively 100 and 25 centimetres were each charged with 100 units of +E. If placed in contact what would be the average electrical density? *Answer.* 1·6.

(13.) What is the ratio of the electrical densities on two spheres whose diameters are respectively 2 and 3 centimetres (1) when their electrical charges are equal, and (2) when they are to one another as 4 : 5?

$$\text{(1.) Since density} = \frac{\text{charge}}{\text{area}} \therefore \frac{\text{density on first sphere}}{\text{,, }\quad\text{second ,,}}$$

$$= \frac{\text{surface of second}}{\text{,, }\quad\text{first}} = \tfrac{9}{4}.$$

(2) Let charge on first sphere = 4 Q; then

$$\text{density on first sphere} = \frac{4\,Q}{4\,\pi} = \frac{Q}{\pi},$$

$$\text{density on second sphere} = \frac{5\,Q}{9\,\pi};$$

$$\therefore \frac{\text{Density on first sphere}}{\text{,, }\quad\text{second ,,}} = \frac{Q}{\pi} \div \frac{5\,Q}{9\,\pi} = \tfrac{9}{5}.$$

(14.) If 100 units of electricity be imparted to each of two spheres, one of which has a radius of 20 centimetres and the other a radius of 5 centimetres, compare the electrical densities on their surfaces. *Answer.* As 1 : 16.

(15.) The semiaxes a, b, c, of an ellipsoidal conductor, are respectively equal to 10, 7, and 5 centimetres. If a charge of 1,000 units be given to the conductor, what will be the electrical density at the extremity of each of the axes?

By a proof which involves the use of the Integral Calculus, it can be shown that if Q represent the electrical charge on an ellipsoid whose semiaxes are a, b, c, then the electrical density at any point whose coordinates are x, y, z, is given by the expression

$$\rho = \frac{Q}{4\,\pi\,a\,b\,c\sqrt{\dfrac{x^2}{a^4}+\dfrac{y^2}{b^4}+\dfrac{z^2}{c^4}}},$$

and, therefore, putting $x = a, y = 0, z = 0$, we have for the electrical density at the extremity of the axis of x

$$\rho_a = \frac{Q}{4\pi bc} = \frac{1{,}000}{4\pi \times 35} = 2{\cdot}27 \text{ nearly.}$$

$$\text{Similarly: } \rho_b = \frac{Q}{4\pi ac} = \frac{1{,}000}{4\pi \times 50} = 1{\cdot}59 \quad ,,$$

$$\rho_c = \frac{Q}{4\pi ab} = \frac{1{,}000}{4\pi \times 70} = 1{\cdot}14 \quad ,,$$

(16.) If the semiaxes a, b, c, were respectively equal to 15, 17, and 19 centimetres, and a charge of 6,000 units is communicated to this ellipsoid, find the electrical density at the extremity of each of the axes.

Answer. 1·48; 1·68; 1·87.

(17.) The longer axis of a prolate spheroid is 100 centimetres, and the shorter axis is 5 centimetres. If an electrical charge be given to this spheroid, compare the electrical densities at the ends of the longer and shorter axes.

In this case $a = 50$; $b = c = 2.5$

$$\therefore \rho_a = \frac{Q}{4\pi bc}; \ \rho_b = \frac{Q}{4\pi ac} \ \therefore \frac{\rho_a}{\rho_b} = \frac{ac}{bc} = \frac{a}{b} = \frac{50}{2{\cdot}5} = \frac{20}{1}.$$

(18.) If the longer axis were 100 centimetres and the shorter 1 centimetre long, compare the electrical densities.

Answer. As 100 : 1.

N.B. This illustrates the increased electrical density at the *points* of a conductor.

(19.) Two insulated metal balls whose radii are respectively 5 centimetres and 0·5 centimetre are connected by a thin wire, and have a certain charge of electricity communicated to them. Compare their electrical densities.

Let Q = total charge; Q_1 and Q_2 the respective charges, and r_1 and r_2 the radii of the two spheres, then since the *capacity* of a sphere is numerically equal to its radius we shall have

$$Q_1 = \frac{Qr_1}{r_1 + r_2} \text{ and } Q_2 = \frac{Qr_2}{r_1 + r_2};$$

and if ρ_1 and ρ_2 represent the densities

$$\rho_1 = \frac{Q_1}{4\pi r_1^2},\ \rho_2 = \frac{Q_2}{4\pi r_2^2};$$

$$\therefore \frac{\rho_2}{\rho_1} = \frac{Q_2}{Q_1} \times \left(\frac{r_1}{r_2}\right)^2 = \frac{r_2}{r_1 + r_2} \times \frac{r_1 + r_2}{r_1} \times \left(\frac{r_1}{r_2}\right)^2 = \frac{r_1}{r_2}$$

$$= \frac{5}{.5} = \frac{10}{1}.$$

(20.) If a charge of 588 units were imparted to the combination, what would be the electrical density on the larger sphere? *Answer.* 1·7.

N.B. The maximum value of ρ in dry air is about 20, for when the electrical density has reached this limit any additional electrical charge will escape into the air in the form of a brush or spark discharge.

(21.) An insulated metal ball of 15 centimetres radius is charged from a machine with 100 units of electricity. Another insulated metal ball of 10 centimetres radius is made to touch the first, and the two are then separated. Compare the electrical densities on their surfaces.

Answer. As 2 : 3.

(22.) A sphere of 10 centimetres diameter is charged until the electrical density is 5. What quantity of electricity is required for this?

By Example 10, $Q = 4\pi r^2 \rho = 4\pi \times 25 \times 5 = 1{,}571$ units.

(23.) What quantity of electricity is required to charge a ball of 6 centimetres diameter so that its surface density may be 6? *Answer.* 679 units.

(24.) Charges of 5 units of electricity are placed at each of the four corners of a square whose side is 10 centimetres. What is the value of the electrical potential *at* the point of intersection of the diagonals?

If a number of electrical charges q_1, q_2, q_3, &c., be placed at a series of points whose distances from a given point are r_1, r_2, &c., then the numerical value of the sum of the series

$\frac{q_1}{r_1}+\frac{q_2}{r_2}+\ldots=\Sigma\frac{q}{r}=\text{V}$ is defined to be the potential *at* that point due to the electrified system.

In the present case $r_1=r_2=\ldots=\frac{5}{\sqrt{2}}$;

$$\therefore \text{V}=\frac{20}{\frac{5}{\sqrt{2}}}=4\sqrt{2}=5{\cdot}66.$$

(25.) If the charges at the several corners were equal to 3, 5, 7, 9 units respectively, what would be the value of the potential *at* the centre? *Answer.* 6·79.

(26.) If charges of 12, 40, and 20 units be placed at the corners A, B, C of a square A B C D whose side is 50 centimetres, calculate the value of the potential *at* the corner D. *Answer.* V = 1·2.

(27.) Charges of 15, 7, 8, and 13 units are placed at the corners A, B, C, D of a square whose side is 5 centimetres. Calculate the value of the electrical potential *at* the middle-point of B C.

In this case $\text{AE}=\text{DE}=5{\cdot}6$ centimetres,
$\text{BE}=\text{EC}=2{\cdot}5$ „

$$\therefore \text{V}=\frac{15+13}{5{\cdot}6}+\frac{7+8}{2{\cdot}5}=5+6=11.$$

(28.) If 10 units of electricity be placed at the middle point of each of the sides of an equilateral triangle whose side is 10 centimetres, find the potential at the centre of the inscribed circle. *Answer.* $6\sqrt{3}$.

(29.) The sides of a triangle are $a=40$, $b=63$, $c=80$ centimetres, and charges of 5, 8, 14·2 units are placed at the corners A, B, C, respectively. Find the potential at the centre of the circumscribing circle.

It is easily proved that R = 13·6 centimetres, and

$$\therefore \; v = \frac{5 + 8 + 14{\cdot}2}{13{\cdot}6} = 2.$$

(30.) The three sides of a triangle are 357, 476, and 595 centimetres respectively and charges of 119 units of electricity are placed at each of the angular points. Find the potential at the centre of the circumscribing circle.

Answer. v = 3.

(31.) The sides of a triangle are 45, 75, and 80 centimetres, and at the angular points are placed electrical charges numerically equal to the lengths of the opposite sides. Find the potential at the point of intersection of the perpendiculars from the angular points on the opposite sides. *Answer.* v = 8·25.

(32.) In Example 26, what amount of work would have to be done against electrical forces to bring a unit of +E from the corner D to the centre of the square?

The work which has to be done against electrical forces to move a unit of +E from one point to another which is at a different potential is numerically equal to the difference between the values of the potential *at* those two points.

But potential at centre of square

$$= \frac{12 + 40 + 20}{\text{semidiagonal}} = \frac{72 \times \sqrt{2}}{50} = 2.04 \text{ nearly.}$$

Potential at D = 1·2 ;

$$\therefore \text{ Work required} = 2{\cdot}04 - 1{\cdot}2 = 0{\cdot}84 \text{ erg.}$$

(33.) In example 28, calculate the work which has to be done against electrical forces to bring a unit of +E from one corner of the triangle to the centre of the inscribed circle. *Answer.* Work required = 5·24 ergs, nearly.

(34.) Calculate the work required to move 5 units of +E from the middle point of B C (in Example 27) to the centre of the square.

Potential at centre $= \frac{43\sqrt{2}}{5} = 12{\cdot}16$; potential at middle point of B C $= 11$;

∴ work required $= 5\,(12{\cdot}16 - 11) = 5 \times 1{\cdot}16 = 5{\cdot}8$ ergs.

(35.) An insulated metal sphere of 1·5 centimetre radius is charged with electricity to the potential 5. How many units of electricity does it contain?

If C represent the capacity of conductor removed from all other conductors, and which is raised by a charge Q from potential zero to potential V, then

$$\text{Q} = \text{C} \times \text{V}, \text{ and in the present case,}$$

$$\text{C} = 1{\cdot}5,\ \text{V} = 5\ ;\ \therefore \text{Q} = 1{\cdot}5 \times 5 = 7{\cdot}5 \text{ units.}$$

(36.) What charge must be given to an insulated metal ball of 5 centimetres diameter, so as to raise its potential from zero to 10? *Answer.* 25 units.

(37.) To what potential must we charge a sphere of one metre in diameter in order that the surface density may be denoted by unity?

$$\text{Let } x = \text{required potential, then } \text{Q} = x \times 50\ ;$$

$$\therefore \rho = \frac{x \times 50}{\pi \times (100)^2} = 1\ ;\ \therefore x = 628{\cdot}3$$

(38.) Supposing that we had a metal ball of 1 centimetre radius, to what potential would it have to be charged that its electrical density might be represented by unity?

Answer. $12\frac{4}{7}$.

(39.) Two spheres whose diameters are one metre and one centimetre are charged with electricity to the same potential. Compare the electrical densities on their surfaces. *Answer.* $d_2 : d_1 = 100 : 1$

(40.) Two insulated metal balls, one of them being 2 centimetres and the other 1·5 centimetre in diameter, were charged by a plate machine to the potential 180. Find the force with which these two balls would repel one another at a distance of 1·5 metre.

In this case $q_1 = 180 \times 1$; $q_2 = 180 \times \cdot75 = 135$;

$$\therefore \text{F} = \frac{q_1 \times q_2}{d^2} = \frac{180 \times 135}{(150)^2} = 1\cdot08 \text{ dyne.}$$

(41.) Two insulated metal spheres whose radii are 0·5 and 0·7 centimetre respectively are placed at a distance of 35 centimetres apart. The first is charged to potential 100, and the second to potential 140. What will be their mutual force of repulsion? *Answer.* 4 dynes.

(42.) How much work has to be expended in charging a sphere of 3 centimetres diameter to potential 10?

While the potential of a conductor is gradually raised from 0 to V the average potential is $\frac{\text{V}}{2}$, and $\therefore$ the whole work done is $\frac{\text{Q V}}{2}$; $\therefore$ in the present case

$$\text{W} = \tfrac{1}{2}\,\text{Q V} = \frac{1\cdot5 \times 10 \times 10}{2} = 75 \text{ ergs.}$$

(43.) A sphere of one decimeter in diameter is charged to potential 100. What amount of work is expended in charging it? *Answer.* 25,000 ergs.

(44.) An uncharged sphere of 5 centimetres radius is placed in a region at potential 5 and an electrical charge of 20 units is imparted to it. What amount of work will be expended in this process?

If the sphere were in a region at zero potential a charge of 20 units would raise its potential to 4, and the work required for this would be $\frac{1}{2}\,\text{Q V} = 20 \times 2 = 40$ ergs. But in the present case the whole charge has first to be raised to potential 5, and this will absorb $20 \times 5 = 100$ ergs.

$\therefore$ Total work required $= 100 + 40 = 140$ ergs.

(45.) What amount of work will have to be expended against electrical force in imparting a charge of 100 units of electricity to a sphere of 10 centimetres radius which is situated in a region at potential 5? *Answer.* 1,000 ergs.

(46.) Three insulated metal spheres, whose radii are 1, 3, 5 centimetres, are placed at a considerable distance apart, and are charged with electricity till their potentials are 1, 3, 5, respectively. They are then connected by thin wires. Find the potential of the system.

If C represent the joint capacity of a system of conductors whose individual capacities are C_1, C_2, C_3, then

$$c = c_1 + c_2 + c_3;$$

and if their initial potentials were V_1, V_2, V_3, their common potential when united by thin wires is

$$v = \frac{v_1 c_1 + v_2 c_2 + v_3 c_3}{c}.$$

In the present case

$$v = \frac{1 + 3 \times 3 + 5 \times 5}{1 + 3 + 5} = \frac{35}{9} = 3\frac{8}{9}.$$

(47.) If the radii of the spheres had been 7, 11, 14 centimetres and their initial potentials 12, 10, 8, respectively, find their common potential after joining them by thin wires. *Answer.* 9·56 nearly.

(48.) The charges on 4 spheres, which are at a considerable distance from each other, are 23, 17, 13, 16 units, and their radii are 5, 7, 3, 8 centimetres, respectively. Find the common potential of the system when they are joined by thin wires. *Answer.* 3.

(49.) If an insulated sphere of 4 centimetres radius were electrically charged to the potential 1,000, calculate the electrical pressure per square centimetre of surface.

The surface density $= \frac{v r}{4\pi r^2} = \frac{v}{4\pi r} = 19{\cdot}89.$

The electrical pressure per square centimetre of surface is

$$4\pi\rho^2 = 4\pi\,(19{\cdot}89)^2 \text{ dynes}$$
$$= \frac{4\pi \times (19{\cdot}89)^2}{981{\cdot}17} \text{ grammes} = 5{\cdot}0785 = 5 \text{ gr. nearly.}$$

(50.) Compare this electrical pressure with that of the atmosphere.

At Greenwich where $g = 981{\cdot}17$ the pressure corresponding to 76 centimetres of mercury at 0°C is $1{\cdot}0138 \times 10^6$ dynes = 1033·3 grammes per square centimetre;

$$\therefore \frac{\text{pressure of atmosphere}}{\text{electrical pressure}} = \frac{1033}{5} = \frac{206{\cdot}6}{1}.$$

(51.) An insulated sphere whose radius is 2 centimetres is charged to the potential 500. Find the electric pressure per square centimetre of surface.

Answer. 5·0785 grammes weight.

(52.) If the sphere in Example 49 had been charged to potential 500, what would have been the electrical pressure per square centimetre?

Answer. The weight of 1·2696 grammes.

EXPERIMENTS WITH THE TORSION BALANCE.

(1.) The balls in a Coulomb's torsion balance were initially in contact. They were then charged, and before the torsion head was moved the movable ball came to rest at an angular distance of 36° from the fixed ball. The distance from the axis of suspension to the centre of the movable ball was 10 centimetres, and the torsional moment of the wire (a silver one) was 2·9 dynes for 1°. Find the charges on each of the balls.

Let P and Q (see fig. 1) be the centres of the two balls; $POQ = \theta$, $OP = r$, and let q = electrical charge on each ball.

The number 2·9 expresses the force, in dynes, which would have to be applied at right angles to the end of a lever arm one centimetre in length, so as to balance the couple due to the torsional force in the wire. Its equivalent for a lever arm of r centimetres and for an angle of 36° is $\frac{36 \times 2{\cdot}9}{r}$.

The mutual force between the two balls is $\frac{q^2}{PQ^2}$ and acts along P Q; hence, taking moments about O, we have

$$\frac{q^2 \cos \frac{\theta}{2}}{4r^2 \sin^2 \frac{\theta}{2}} = \frac{2{\cdot}9 \times 36}{r},$$

whence,

$$q = \sin \frac{\theta}{2} \sqrt{\frac{4r \times 2{\cdot}9 \times 36}{\cos \frac{\theta}{2}}} = \sin 18^\circ \sqrt{\frac{40 \times 2{\cdot}9 \times 36}{\cos 18^\circ}}$$
$$= 20{\cdot}48.$$

(2.) The torsion head was then twisted through 126°, and the angular separation of the balls was reduced to 18°. What was the electrical force between the balls?

The angle through which the wire was twisted = 126° + 18° = 144°.

The corresponding torsional force at end of balance arm = $\frac{144 \times 2{\cdot}9}{10}$ dynes.

The component of the mutual force between the balls resolved at right angles to the beam = F cos 9°; whence,

$$F \cos 9^\circ = \frac{144 \times 2{\cdot}9}{10}; \therefore F = \frac{14{\cdot}4 \times 2{\cdot}9}{\cos 9^\circ} = 42{\cdot}28 \text{ dynes.}$$

(3.) The torsion head was then twisted through a further additional angle of 441°, and the angular separation of the balls was thereby reduced to 8° 30′. What was the electrical force between the balls?

Answer. 167·35 dynes.

(4.) Suppose that in Example 1, before the torsion head was moved, the movable ball came to rest at an angular distance of 20° from the fixed one, what would have been the charge on each ball?

We have

$$q = \sin\frac{\theta}{2}\sqrt{\frac{4r \times 2·9 \times \theta^\circ}{\cos\frac{\theta}{2}}} = \sin 10^\circ\sqrt{\frac{40 \times 2·9 \times 20}{\cos 10^\circ}}$$

$$= 8·428 \text{ units.}$$

(5.) If instead of the angular separation being 20° it had been 15° in one case and 12° in another, what would have been the charges on each of the balls?

Answer. 5·468 and 3·912 units.

(6.) Both balls of a torsion balance are in contact at the beginning of an experiment. A charge is then shared equally between them, they repel each other, and in order to bring their angular separation to 10° the torsion head has to be twisted through 50°. The length of the balance beam being 20 centimetres, and the torsional co-efficient of the wire for 1° being 0·076 dyne, calculate the charge on each ball. *Answer.* 1·179 unit.

(7.) Had an error of 1° in excess been made in reading off the angle of torsion, what would have been the consequent error in the calculated value of the charge?

Answer. 0·0107 unit.

(8.) In another experiment with the balance described in Example 1, the torsion head was turned so that when the balls were not electrified their angular separation was 10°. The movable ball was then charged with 10 units of + E, and a certain charge was imparted to the fixed ball, when it was found that in order to reduce the angular separation of the balls to 10° the torsion head had to be twisted through 150°. What was the charge on the fixed ball?

Let q_1 and q_2 be the charges on the fixed and movable balls respectively, and let $2r$ = length of balance beam; then as in Example 1 we have

$$\frac{q_1 q_2}{4r^2 \sin^2 5^\circ} \cos 5^\circ = \frac{150 \times 2{\cdot}9}{r};$$

$$\therefore q_1 = \frac{4r \sin^2 5^\circ \times 150 \times 2{\cdot}9}{q_2 \times \cos 5^\circ} = \frac{40 \times 150 \times 2{\cdot}9 \times \sin^2 5^\circ}{10 \times \cos 5^\circ}$$

$$= 13{\cdot}268 \text{ units.}$$

(9.) If the charge on the movable ball were always 10 units, and when the fixed ball received different charges the angles through which the torsion head had to be moved so as to reduce the angular separation of the balls to 10° were 170°, 220°, and 367°, what were the charges on the fixed ball in each case?

Answer. 15·04, 19·46, 32·46 units approximately.

(10.) In order to compare the electrical surface densities at two points A and B of an insulated metallic surface which had been positively electrified, the movable ball of a torsion balance was slightly charged with +E and was then opposite to the zero of the scale. The point A was then touched with a little *proof plane*, which was then inserted in the balance at the zero, but so as not to touch the movable ball, which was repelled and came to rest at an angular distance of 13° from the zero. The proof plane was then removed, discharged; B was touched with it, and the above process was repeated when the angular deflection of the movable ball was 10°. Compare the electrical densities at A and B.

Let f = force between the proof plane and the movable ball for a given charge at the unit distance; then if F = force at a distance for which the angular separation of the balls is θ, and if $2l$ = length of balance beam, by the method of Example 1 we have

$$F = \frac{f \cos \frac{\theta}{2}}{4l^2 \sin^2 \frac{\theta}{2}}.$$

Let ϕ = total number of degrees of torsion on the wire in the position of equilibrium, then the force of torsion =

$m\phi$, where m is a coefficient whose value depends upon the particular wire employed, and is numerically equal to the force corresponding to a torsion of 1°. Since there is equilibrium we must have

$$m\phi = \frac{f\cos\frac{\theta}{2}}{4l^2\sin^2\frac{\theta}{2}}, \text{ or } \frac{f}{4l^2} = m\phi\sin\frac{\theta}{2}\tan\frac{\theta}{2}.$$

In the present case it is evident that we have to compare the forces at unit distance, and ∴ if f_1 and f_2 be the forces corresponding to A and B we shall have

$$\frac{f_1}{f_2} = \frac{\phi_1\sin\frac{\theta}{2}\tan\frac{\theta}{2}}{\phi_2\sin\frac{\theta'}{2}\tan\frac{\theta'}{2}} = \frac{13\sin 6\frac{1}{2}^\circ\tan 6\frac{1}{2}^\circ}{10\sin 5^\circ\tan 5^\circ} = \frac{11}{5} \text{ nearly.}$$

(11.) The fixed and movable balls of a torsion balance are of equal size, and when each is charged with one unit of +E it is found that their angular distance apart is 18°. They are again charged, and in order to reduce their angular separation to 18° it is found necessary to twist the torsion head through 126°. What is the charge on each ball? *Answer.* $2\sqrt{2}$ units.

(12.) If with a given charge the movable ball is at an angular distance of 90° from the fixed ball when the twist on the wire is 100°, through how many degrees will the torsion head have to be turned, so as to bring the movable ball to within 60° of the fixed ball? *Answer.* 130°.

(13.) In an experiment with the torsion balance the balls were at an angular distance of 20° when the twist of the torsion head was 250°. Owing to the escape of the electricity the distance diminished, and after the lapse of a minute the twist of the torsion head had to be reduced 6°, so as to bring the balls again to an angular distance of 20° apart. What was the loss of electricity per minute?

At the commencement of interval the force

$$= m\,(250 + 20) = m \times 270.$$

At the end of interval the force

$$= m\,(244 + 20) = m \times 264.$$

∴ Mean electrical force during interval

$$= m\,\frac{270 + 264}{2} = m \times 267$$

∴ Loss of electricity in one minute

$$= \frac{6}{267} = \frac{1}{44}, \text{ nearly.}$$

(14.) In another experiment the electrical repulsive force was initially proportional to an angle of torsion equal to 270°. After the lapse of three minutes the torsion had to be reduced by 60°, so as to bring the balls to their initial distance apart. Find the loss of electricity per minute.

Answer. $\frac{1}{12}$

(15.) Both balls of the torsion balance were initially in contact. A charge was then imparted to them, and they repelled each other, but by twisting the torsion head through 140° they were brought to a distance of 20° apart. They gradually approached each other, and after three minutes the torsion had to be reduced by 30° in order to separate them to a distance of 20°. What was the loss of electricity per minute? *Answer.* $\frac{1}{14 \cdot 5}$

(16.) To determine the ratio of the electrical densities at two points A and B of a charged conductor these two points were touched simultaneously by two small equal proof spheres, p and q. When p was placed in the balance a torsion of 55°·5 was required, so as to reduce the separation of the balls to 30°, and when q was substituted for p a torsion of 293°·5 was required for the same purpose, 3·1 minutes having elapsed between the two observations. The torsion was then reduced by 20°, and in 3·2 minutes the balls were again 30° apart. Compare the electrical densities at A and B.

In 3·2 minutes the loss of charge by the fixed ball was represented by 20° of torsion. If the loss were simply proportional to the time, then in the interval between the times when p and q were inserted in the balance, the loss of charge by the fixed ball would have been $\frac{3·1}{3·2} \times 20 = 19°·4$. But since the loss is also proportional to the electric density on the fixed ball, we may, without sensible error, assume that the electric density on it during the first interval (3·1 minutes) is to its electric density during the second interval (3·2 minutes) as 293·5 : 273·5, and ∴ the loss of charge between the two observations $= 19·4 \times \frac{293·5}{273·5} = 20°·8$.

Hence *at the moment* when the electrical charge at A was being measured the charge at B corresponded to 293°·5 × 20°·8 = 314°·3 of torsion.

$$\therefore \frac{\text{Electrical density at B}}{\text{Electrical density at A}} = \frac{314·3}{55·5} = \frac{5·66}{1} \text{ nearly.}$$

(17.) Two points, P and Q, on a charged conductor, were touched at the same time by two little proof-spheres, p and q. On putting p into the balance a torsion of 67° was required to reduce the separation of the balls to 20°, and on substituting q for p a torsion of 369° was required for the same purpose, the interval between the two observations being 3·7 minutes. The torsion was then reduced by 20°, and after an interval of 2·5 minutes the separation of the balls was again 20°. Compare the electrical densities at P and Q. *Answer.* $\frac{\text{Density at Q}}{\text{Density at P}} = \frac{6}{1}$ nearly.

(18.) A small pith ball covered with gold leaf was fastened to one end of a long glass needle which was capable of rotating about a vertical axis through its centre. Two small insulated metal spheres, A and B, charged with +E were placed so that their centres were on the circumference of the circle described by the pith ball, which had also a small positive charge given to it. In the position of equilibrium the angular distances of the small ball from the spheres A and B were 20° and 45° respectively. Compare the electrical charges on A and B.

Let q = charge on C and q_1, q_2 the charges on A and B (see fig. 2).

The force between B and C $= \dfrac{q\,q_2}{\text{B C}^2}$ and acts along B C.

∴ Moment of the force due to B tending to twist O C

$$= \frac{q\,q_2}{4\,r^2 \sin^2 \frac{\theta_2}{2}} \cos \frac{\theta_2}{2}.$$

Moment of the force due to A tending to twist O C

$$= \frac{q\,q_1}{4\,r^2 \sin^2 \frac{\theta_1}{2}} \cos \frac{\theta_1}{2},$$

And when there is equilibrium these moments must balance each other, and then

$$\frac{q_2 \cos \frac{\theta_2}{2}}{\sin^2 \frac{\theta_2}{2}} = \frac{q_1 \cos \frac{\theta_1}{2}}{\sin^2 \frac{\theta_1}{2}};\ \therefore \frac{q_1}{q_2} = \frac{\tan \frac{\theta_1}{2} \sin \frac{\theta_1}{2}}{\tan \frac{\theta_2}{2} \sin \frac{\theta_2}{2}}$$

$$= \frac{\tan 10^\circ \sin 10^\circ}{\tan 22\tfrac{1}{2}^\circ \sin 22\tfrac{1}{2}^\circ} = \frac{100}{518}.$$

(19.) If the angular distances of the movable sphere from A and B when in equilibrium were 30° and 60° respectively, and the charge on the sphere A were 2 units, what would be the charge on B? *Answer.* 8·3 units, nearly.

Note.—On the difficulties attending the employment of the torsion balance as an instrument for exact research, the student should consult Clerk-Maxwell's 'Electricity and Magnetism,' vol. i. art. 43.

STATICAL INDUCTION.

(1.) A metal ball of 3 centimetres radius is placed in a space whose potential is 6, and the ball is then touched and removed. What will now be its free charge?

On connecting the ball for an instant with the earth $3 \times 6 = 18$ units of $+\mathrm{E}$ will escape to the earth and leave the ball charged with 18 units of $-\mathrm{E}$.

(2.) A sphere was placed in a region at potential 5 and was then brought to zero potential. On insulating and removing it, the sphere was found to have acquired a charge of 25 units of $-\mathrm{E}$. What was its radius?

Answer. 5 centimetres.

(3.) A sphere of 10 centimetres radius was placed in a space at potential 3 and then charged until its potential was 7. What was its charge?

Since the charge raises the potential from 3 to 7,

$$Q = 10 \times (7-3) = 40 \text{ units.}$$

(4.) A sphere whose radius was 10 centimetres was placed in a space at potential 7. It was then brought to potential 5, insulated, and removed. What would now be its charge? *Answer.* $10 \times 2 = 20$ units of $-\mathrm{E}$.

(5.) A metal ball of 5 centimetres radius was charged with 20 units of $+\mathrm{E}$, and then placed in a region at potential 7. What would then be the potential of its charge?

Answer. $4 + 7 = 11$.

(6.) Two insulated spheres, each of 4 centimetres radius, were connected by a long thin wire, and one of them was placed in a space at potential 5 and the other in a space at potential 7. What will be the potential and the charge on each sphere?

Answer. Potential = 6. Charges +4 and −4 units.

(7.) A sphere of 5 centimetres radius is suspended within a hollow concentric uninsulated sphere of 10 centimetres radius. Find the charge which it will take if electrified to the potential 4.

If r_1 r_2 be the radii of the inner and outer surfaces, then the capacity of such a spherical condenser is

$$C = \frac{r_1 r_2}{r_2 - r_1};$$

and if Q = total charge and V = the potential, Q = C × V, and

$$\therefore Q = \frac{r_1 r_2}{r_2 - r_1} \times V = \frac{5 \times 10}{5} \times 4 = 40 \text{ units.}$$

(8.) If the radii of the inner and outer spherical surfaces were 5 and 6 centimetres respectively, and the potential of the insulated sphere were raised to 5, what would be its charge? *Answer.* 150 units.

(9.) To what potential must the inner sphere be raised, if the charge is to be 252 units and the radii of the inner and outer surfaces are 7 and 9 centimetres? *Answer.* 8.

(10.) An *air-condenser* was formed of two circular metal plates of 5 centimetres radius, their distance apart being 0·5 centimetre. The collecting plate was charged to potential 4. What was its charge?

Let A = area of collecting plate in square centimetres and t = thickness of the layer of dry air between it and the condensing plate; then if C represent the capacity of the condenser we have

$$C = \frac{A}{4\pi t} \text{ and } Q = C \times V,$$

$$\therefore Q = 4 \times \frac{\pi \times 25}{4\pi \times \cdot 5} = 50 \text{ units.}$$

(11.) What would have to be the potential of the source so that the charge on the collecting plate may be 100 units, if its radius is 10 centimetres and its distance from the condensing plate 1·5 centimetre? *Answer.* 6.

(12.) Compare the electrical capacity of a condenser formed of two equal circular discs of one decimetre in diameter with that of a spherical condenser of the same diameter, the dielectric being a stratum of air one centimetre thick in each case. *Answer.* As 5 : 24.

(13.) A sphere of 10 centimetres diameter was charged with electricity to the potential 5. A disc of 10 centimetres diameter which had a condensing plate at a distance of 0·5 centimetre, the dielectric being air, was also charged till its potential was 5. Compare the charges on the disc and sphere. *Answer.* $\frac{\text{Charge on disc}}{\text{Charge on sphere}} = \frac{2}{5}$.

(14.) A sphere of 20 centimetres diameter was charged to the potential 5. A metal disc 10 centimetres in diameter had a condensing plate which was separated from it by a layer of dry air. What must be the distance between the two plates, so that when charged to the potential 5 the charge on the disc should be equal to that on the sphere?

Answer. $\frac{5}{8}$ centimetre.

(15.) A disc of 15 centimetres diameter separated from its condensing plate by a sheet of mica ·1 centimetre thick, whose specific inductive capacity was 5, was charged to the potential 2. What must be the diameter of a sphere which, when charged to the same potential, had a charge on it equal to that on the disc? *Answer.* 1406·25 centimetres.

(16.) The leaves of a gold-leaf electroscope, insulated from each other, were connected by thin wires to two parallel insulated metal discs A and B at zero potential. A positively charged disc C was then placed midway between A and B, and the gold leaves remained vertical, but on introducing a plate of shellac 3 centimetres thick between A and C, it was necessary to withdraw the plate A by 1·9 centimetres further from C so as to restore the verticality of the gold leaves. What was the specific inductive capacity of the shellac?

Let x = S.I.C. of shellac; T = thickness of shellac plate, and t the space through which A is withdrawn so as to restore equilibrium. The shellac plate is evidently equivalent to a layer of air of $\frac{1}{x}$ times its inductive capacity, and therefore we shall have

$$T = t + \frac{T}{x}, \text{ whence } x = \frac{1}{1 - \frac{t}{T}} = \frac{1}{1 - \frac{1\cdot9}{3}} = 2\cdot\dot{7}\dot{2}.$$

$$= 2\cdot73, \text{ nearly.}$$

(17.) A plate of wax 1·7 centimetre thick was then substituted for the above shellac plate, and the plate A had now to be withdrawn through a distance of 0·79 centimetre, so as to restore equilibrium. Calculate from this experiment the S.I.C. of the wax. *Answer.* 1·86.

(18.) An air condenser was formed of two metal discs, and its electrostatic capacity was determined in the usual manner. A plate of paraffin, 3 centimetres thick, was then introduced between the discs, and the condensing plate had to be withdrawn through a distance of 1·7 centimetre so as to make the capacity of the condenser the same as before. Find from these data the S.I.C. of the paraffin.

Answer. 2·3.

(19.) A plate of sulphur 2·5 centimetres thick was next introduced between the discs, and the requisite displacement of the condensing plate was then 1·9 centimetre. Find from this the S.I.C. of the sulphur.

Answer. 4·2 nearly.

(20.) If the dielectric in Example 8 had been of shellac, whose S.I.C. was 2, what would have been the charge?

If the dielectric be not *air* but some substance whose inductive capacity relatively to air is K, then the capacity of the condenser will be

$$c = \frac{r_1\, r_2}{r_2 - r_1}\,\text{K}, \text{ and } \therefore\ Q = C \times V = 150 \times 2 = 300 \text{ units.}$$

(21.) Find the electrostatic capacity of a spherical condenser, the radius of the inner coating being 7 centimetres, that of the outer coating 7·3 centimetres, and the specific inductive capacity of the insulator 1·9. *Answer.* 323·6.

(22.) What is the *condensing power* of an air condenser formed of an insulated disc of 6 centimetres radius, with a condensing plate at a distance of 0·3 centimetre from it?

The *condensing power* is the ratio of the charge which the collecting plate can take when the condensing plate is present to that which it would take if the second plate were absent, and it were electrified to the same potential.

Now the capacity of a disc *with* condensing plate

$$= \frac{A}{4\pi t} = \frac{\pi r^2}{4\pi t} = \frac{r^2}{4t}$$

and the capacity of a disc *without* condensing plate

$$= \frac{2r}{\pi}.$$

$$\therefore \text{ ‘Condensing power’} = \frac{r^2}{4t} \times \frac{\pi}{2r} = \frac{\pi r}{8t} = \frac{\pi \times 6}{8 \times {\cdot}3}$$
$$= 7{\cdot}854 = 7{\cdot}9 \text{ nearly.}$$

(23.) Calculate the value of the ‘condensing power’ of an insulated sphere of 5 centimetres radius surrounded by a concentric uninsulated sphere of 5·5 centimetres radius.

Answer. 11.

(24.) An insulated metal disc of 8 centimetres radius was charged to the potential 7. A large uninsulated metal disc was then placed parallel to it at a distance of 1·5 centimetres. What was then the potential of the charge on the collecting plate?

$$\text{Let } \left.\begin{array}{l} v_1 = \text{original potential} \\ v_2 = \text{final} \quad ,, \end{array}\right\} \left.\begin{array}{l} c_1 = \text{original capacity} \\ c_2 = \text{final} \quad ,, \end{array}\right\}$$

$$Q = \text{charge.}$$

Then $Q = v_1 c_1 = v_2 c_2$ and $\therefore v_2 = \frac{v_1 c_1}{c_2}$

$$= v_1 \times \frac{2r}{\pi} \times \frac{4\pi t}{\pi r^2} = v_1 \times \frac{8t}{\pi r} = \frac{7 \times 8 \times 1\cdot 5}{8\pi} = 3\cdot 34.$$

(25.) An insulated sphere of 10 centimetres radius was charged to the potential 4, and was then inclosed in an uninsulated concentric sphere of 12 centimetres radius. What was then the potential of the charge on the inner sphere? *Answer.* $\frac{2}{3}$.

(26.) An insulated sphere of 5 centimetres radius charged to potential 10 is connected for an instant to an insulated metal disc of 5 centimetres radius. After removal of the sphere a condensing plate is brought up to a distance of 0·5 centimetre from the charged plate. Find the potential of this plate.

Charge on sphere = 5 × 10 = 50 units. If V_1 = potential of charge on disc after the contact

$$v_1 = \frac{50}{5 + \frac{10}{\pi}} = \frac{50\pi}{5\pi + 10.}$$

Also if V_2 = final potential of charge on collecting plate,

$$v_2 = v_1 \times \frac{8 \times \cdot 5}{5\pi} = \frac{50\pi}{5\pi + 10} \times \frac{\cdot 8}{\pi} = \frac{8}{\pi + 2} = 1\cdot 56, \text{ nearly.}$$

(27.) The distance between the plates of an air condenser was one millimetre. The area of the collecting plate was 100 square centimetres, and it was connected by a long thin wire to an insulated sphere of 5 centimetres radius. If a charge of 1000 units be imparted to the system, what would be the respective charges on the sphere and the condenser?

Let C_1 = capacity of sphere = $r = 5$

$$C_2 = \quad " \quad \text{condenser} = \frac{100}{4\pi \times \cdot 1} = \frac{250}{\pi}.$$

Then if Q = total charge, Q_1 = charge on sphere, and Q_2 = charge on condenser.

$$Q_1 = Q \frac{C_1}{C_1 + C_2} = \frac{5 \times 1{,}000}{5 + \frac{250}{\pi}} = 59\cdot 12, \text{ nearly.}$$

$$Q_2 = Q \frac{C_2}{C_1 + C_2} = \frac{250 \times 1{,}000}{5\pi + 250} = 940\cdot 88 \text{ units.}$$

(28.) What would have been the charge on the sphere if its radius were 7 centimetres, the area of the collecting plate 240 square centimetres, its distance from the condensing plate 0·2 centimetre, and the total charge 1000 units?

Answer. 68·3 units.

(29.) A thin insulated metal disc of 8 centimetres radius is charged with 25 units of electricity, and then an uninsulated condensing plate is brought up to a distance of 0·2 centimetre from it. What was the potential of the charge on the disc before and after the approach of the condensing plate?

Let C_1 and C_2 be the respective capacities in the two cases; then

$$C_1 = \frac{2r}{\pi}; \; C_2 = \frac{\pi r^2}{4\pi t} = \frac{r^2}{4t};$$

$$\therefore V_1 = \frac{Q}{C_1} = \frac{25 \times \pi}{2r} = \frac{25 \times \pi}{16} = 4\cdot 9$$

$$V_2 = \frac{Q}{C_2} = \frac{25 \times 4 \times .2}{64} = 0\cdot 31$$

(30.) Two metal discs, A and B, each of 10 centimetres radius, were connected by a long thin wire. The disc B was separated from a condensing plate by a plate of glass one millimetre thick, and a charge of 100 units was given to the system. If the S.I.C. of glass be 2, calculate the charges on A and B respectively.

Let $C_1 =$ capacity of A, then $C_1 = \frac{2r}{\pi}$

$$C_2 = \quad ,, \quad B \qquad C_2 = 2\,\frac{r^2}{4 \times \cdot 1} = \frac{r^2}{\cdot 2}$$

and if $Q =$ total charge, and Q_1 Q_2, the partial charges

$$Q_1 = Q \times \frac{C_1}{C_1 + C_2} = \frac{100 \times \frac{2r}{\pi}}{\frac{2r}{\pi} + 5r^2} = \frac{200}{159 \cdot 08} = 1 \cdot 26\ ;$$

$$Q_2 = Q \times \frac{C_2}{C_1 + C_2} = \frac{500\pi r}{2 + 5\pi r} = \frac{15708}{159 \cdot 08} = 98 \cdot 74.$$

(31.) If the discs were each of 7 centimetres radius and the disc B separated from its condensing plate by a plate of shellac 3 millimetres thick whose S.I.C. is 2, find the charge on each plate when a charge of 100 units is imparted to the whole system.

Answer. Charge on A = 5·17 units. Charge on B = 94·83 units.

(32.) A condenser was formed of two equal rectangular metal plates whose lengths were 70 centimetres and their widths 20 centimetres. They were separated from each other by a sheet of indiarubber one millimetre thick whose specific inductive capacity was 2·8. What was the capacity of this condenser?

Capacity $= K \times \frac{A}{2\pi t} = 2 \cdot 8 \times \frac{20 \times 70}{2\pi \times 0 \cdot 1} = 6238 \cdot 9$ electrostatic units.

(33.) If the dielectric had been of guttapercha whose S.I.C. was 4·2, and its thickness 0·5 centimetre, what must have been the area of each plate so that the condenser might have the same electrostatic capacity as the one mentioned in the last example?

Answer. 4666·7 square centimetres.

(34.) Two discs of tinfoil, each of 10 centimetres radius, were fixed on opposite sides of a disc of paraffin 0·7 centimetre thick, and whose S.I.C. was 1·98. One of the discs was then connected to an electrical source whose potential was 4, and the other with the ground. Calculate the charge.

Answer. 283 units, nearly.

(35.) Two equal sheets of tinfoil, A and B, were pasted on opposite sides of a glass plate, and one of them was connected with the ground and the other received an electrical charge of 20 units. If a unit of positive electricity on the collecting plate can hold 0·99 unit of negative on the condensing plate, find what portion of the whole charge on the collecting plate would be removed if it were connected with the ground, the condensing plate being at the same time insulated.

Let c = capacity of collecting plate (A) for bound charge and c_1 the free capacity of each of the sheets of tinfoil. Then if v = potential of charge on A at first,

Charge on collecting plate A $= Q_1 = (c + c_1)\,v$

„ „ condensing plate B $= Q_2 = -c\,v$

If now B be insulated and A put to earth, the charge on B will be divided as follows :—

$$\text{Bound charge on B} = \frac{c}{c + c_1} Q_2 = -\left(\frac{c}{c + c_1}\right)^2 Q_1$$

$$\text{Free ,, ,,} = \frac{c_1}{c + c_1} Q_2 = -\frac{c\,c_1}{(c + c_1)^2} Q_1$$

$$\therefore \text{ Potential of B} = \frac{Q_2}{c + c_1} = \frac{-c\,v}{c + c_1}$$

$$\text{and charge on A} = -\frac{c\,Q_2}{c + c_1} = \frac{c \times c\,v}{c + c_1} = \left(\frac{c}{c + c_1}\right)^2 Q_1$$

$$\therefore \text{ Loss of charge} = Q_1 \left\{1 - \left(\frac{c}{c + c_1}\right)^2\right\}$$

But when $(c + c_1)\,v = 1$, then $-c\,v = -{\cdot}99$.

$$\therefore \frac{c}{c + c_1} = {\cdot}99.$$

$\therefore$ Loss of charge $= 20\{1 - ({\cdot}99)^2\} = 0{\cdot}398$ unit, and this was the amount of *free* charge on the plate A.

(36.) What was the numerical value of the condensing force?

The condensing force being the ratio of the total charge to the free charge is in the present case

$$= \frac{20}{\cdot 398} = 50\cdot 26 = 50, \text{ nearly.}$$

(37.) A condenser was constructed like the one described in Example 35, and, after charging, the collecting plate was found to contain 4·86 units of free positive electricity. After removing the free electricity from the collecting plate the condensing plate was found to contain 4·23 units of free negative electricity. Find what quantity of negative electricity was held bound by a unit of +E on the collecting plate? *Answer.* 0·87 unit, nearly.

(38.) A circular disc of 10 centimetres radius has a condensing plate at a distance of 0·2 centimetre, and a charge of 100 units is communicated to it. What portion of this charge is free, and what bound?

The capacity of a disc of radius r when freely electrified is $\frac{2r}{\pi}$, but when this disc forms the collecting plate of a condenser the free charge exists only on that side of the collecting plate which is remote from the condensing plate. Hence its free capacity will be $\frac{r}{\pi}$.

The capacity for bound charge $= \frac{\pi r^2}{4\pi t} = \frac{r^2}{4t}$.

$$\therefore \text{ Free charge} = 100\frac{\frac{r}{\pi}}{\frac{r}{\pi}+\frac{r^2}{4t}} = \frac{4t\times 100}{4t+r\pi} = \frac{80}{32\cdot 216} = 2\cdot 48;$$

$$\text{Bound charge} = \frac{100\times\frac{r^2}{4t}}{\frac{r}{\pi}+\frac{r_1}{4t}} = \frac{1000\pi}{32\cdot 216} = 97\cdot 52.$$

(39.) If a charge of 100 units be imparted to an air condenser whose collecting plate has a radius of 5 centimetres, and is at a distance of one millimetre, find the amount of free and bound charge.

Answer. Free charge = 2·48 ; Bound charge = 97·52 units.

(40.) If the collecting plate be then withdrawn to a distance of one centimetre from the condensing plate, what will be the quantities of free and bound charge?

Answer. Free charge = 20·3 units ; Bound charge = 79·7 units.

(41.) Suppose the collecting plate were still further withdrawn, so that its distance from the condensing plate were 2 centimetres, what would be the quantities of free and bound charge? *Answer.*
Free charge = 33·7 units.
Bound charge = 66·3 „

(42.) The plates of a condensing electroscope were one decimetre in diameter, and were initially separated from each other by a very thin layer of shellac varnish. The lower plate was first earth-connected, and the upper plate had a charge of 100 units imparted to it ; the lower plate was then insulated and the upper one lifted off. Find the potential of the charge on the lower plate.

In this case the whole charge is *bound* at first.

$$\therefore \text{ Potential of lower plate} = -\frac{100}{\frac{2r}{\pi}} = -10\pi = -31{\cdot}416.$$

(43.) The internal radius of a Leyden jar is 6 centimetres, and the height of coated surface 18 centimetres, the average thickness of glass being 0·16 centimetre, and its S.I.C. 1·76. The radius of the knob is 1·6 centimetre. If a charge of 1000 units be imparted to the jar, find what portion of it is free and what bound.

Let h = height of coated surface ; r = inner radius ; t = average thickness of the glass, then

$$\text{Area of inner coating} = 2\pi r h + \pi r^2 = \pi r(r+2h);$$

$$\therefore \text{Capacity for bound charge} = C_1 = \kappa \frac{A}{4\pi t} = \kappa \frac{r(r+2h)}{4t}$$

$$= \frac{1{\cdot}76 \times 6 \times 42}{4 \times {\cdot}16} = 693.$$

The free charge is only that on the knob, and if the radius of the knob be r', capacity for free charge $= C_2 = r' = 1{\cdot}6$.

$$\therefore \text{Bound charge} = 1000 \times \frac{C_1}{C_1 + C_2} = \frac{1000 \times 693}{694{\cdot}6}$$

$$\doteq 997{\cdot}7.$$

$$\text{Free charge} = 1000 \times \frac{C_2}{C_1 + C_2} = \frac{1{\cdot}6 \times 1000}{694{\cdot}6}$$

$$= 2{\cdot}3 \text{ units.}$$

(44.) Two Leyden jars were taken, the internal radii of their bases being 5 and 10 centimetres respectively, while the heights of their coated surfaces were 20 and 40 centimetres. The dielectric was glass, which, in the smaller jar, was 0·3 and in the larger one 0·4 centimetre thick, its S.I.C. being 1·9. The jars were both charged to the potential 8. Compare the charges on the two jars.

Answer. As 1 : 3.

(45.) The radius of the base of the inside of a Leyden jar is 4 centimetres, and the height of coated surface is 10 centimetres. The radius of the knob is one centimetre. The dielectric is glass, one millimetre thick, and of S.I.C. 1·9, and the jar is charged to potential 5. What is the total charge? *Answer.* 2285 units.

(46.) If the large jar in Example 44 be charged to potential 4 and the smaller one to potential 6, and the two jars be then placed on the table and their knobs connected, what will be their common potential?

Let V_1 and V_2 be the potentials of the two jars before contact, V their common potential after contact, and C_1, C_2 their capacities, then it is obvious that

$$\text{v}\ (\text{c}_1 + \text{c}_2) = \text{v}_1\,\text{c}_1 + \text{v}_2\,\text{c}_2,$$

$$\therefore \quad \text{v} = \frac{\text{v}_1\,\text{c}_1 + \text{v}_2\,\text{c}_2}{\text{c}_1 + \text{c}_2} = \frac{\text{v}_1 + \text{v}_2\,\frac{\text{c}_2}{\text{c}_1}}{1 + \frac{\text{c}_2}{\text{c}_1}}.$$

$$\text{But } \text{c}_1 = k\,\frac{r_1\,(r_1 + 2\,h_1)}{4\,t_1};\ \text{c}_2 = k\,\frac{r_2\,(r_2 + 2\,h_2)}{4\,t_2};$$

$$\therefore \frac{\text{c}_2}{\text{c}_1} = \frac{r_2 + 2\,h_2}{r_1 + 2\,h_1} \times \frac{r_2\,t_1}{r_1\,t_2} = \frac{10 + 80}{5 + 40} \times \frac{10 \times {\cdot}3}{5 \times {\cdot}4} = 3;$$

$$\therefore \ \text{v} = \frac{6 + 4 \times 3}{1 + 3} = \frac{18}{4} = 4{\cdot}5.$$

(47.) If the jar in Example 43 were charged to potential 4 and the jar in Example 45 to potential 5, and their knobs were then connected, their outer coatings being put to earth ; find their common potential.

Answer. 4·4, nearly.

(48.) Find the amount of work which would have to be expended in charging the two jars in Example 46 to their respective potentials.

$$\text{Since } \text{w} = \frac{\text{v}^2 \times \text{c}}{2} \text{ we have}$$

$$\text{w}_1 = \frac{k\,(r_1 + 2\,h_1)}{8\,t_1} \times \text{v}_1^2 = \frac{1{\cdot}9 \times 45 \times 36}{8 \times {\cdot}3} = 1{,}282{\cdot}5 \text{ ergs.}$$

$$\text{w}_2 = \frac{k\,(r_2 + 2\,h_2)}{8\,t_2} \times \text{v}_2^2 = \frac{1{\cdot}9 \times 90 \times 16}{8 \times {\cdot}4} = 855 \text{ ergs.}$$

(49.) Had the two jars been placed on a table with their knobs in contact, and been then charged up to the potential 4·5, what amount of work would have been required for this ?

Answer. 1845·5 ergs.

N.B. The student should notice that the work required is less when the jars are in contact than when they are not.

(50.) Three condensers, A, B, C, were made by pasting discs of tinfoil upon opposite sides of a sheet of glass. The radii of the discs were 3, 5, and 7 centimetres respectively, and the corresponding thicknesses of glass were 1, 2, and 2·5 millimetres. The condensers were then mounted so as to be charged by *cascade*, the collecting plate of A being raised to potential 8, while the condensing plate of C was put to earth. Taking the S.I.C. of the glass as 1·9, find the charge on the collecting plate of A.

Let $V_1\ V_2\ V_3$ represent the potentials of the collecting plates of A, B, C.

Let $C_1\ C_2\ C_3$ represent their capacities for bound charge.

Let $Q_1\ Q_2\ Q_3$ represent their charges.

Then
$$\frac{1}{C_1} = \frac{4t_1}{kr_1^2} = \frac{4 \times {\cdot}1}{1{\cdot}9 \times 9} = {\cdot}02339,$$
$$\frac{1}{C_2} = \frac{4t_2}{kr_2^2} = \frac{4 \times {\cdot}2}{1{\cdot}9 \times 25} = {\cdot}01684,$$
$$\frac{1}{C_3} = \frac{4t_3}{kr_3^2} = \frac{4 \times {\cdot}25}{1{\cdot}9 \times 49} = {\cdot}01074\ ;$$
$$\therefore \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = {\cdot}05097.$$

Also $Q_1 = C_1(V_1 - V_2)$; $Q_2 = C_2(V_2 - V_3)$; $Q_3 = C_3 V_3$;
$$\therefore \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} = V_1\ ;$$

Since we are neglecting the free charges the values of Q_1, Q_2, Q_3 must be equal, and $\therefore$
$$Q_1\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = V_1\ ;$$
$$\therefore Q_1 = \frac{V_1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{8}{{\cdot}05097} = 157 \text{ units, nearly.}$$

(51.) Had the condensing plate of A been directly connected with the earth, what would have been the charge on A? *Answer.* 342 units.

(52.) Three exactly equal condensers were constructed as in Example 50, the radii of the tinfoil discs being 10 centimetres, the thickness of glass ·14 centimetre, and its S.I.C. 1·9. If these condensers be joined up in 'cascade' and the charge on the collecting plate of the first be 1,000 units, what is its potential? *Answer.* 8·84, nearly.

(53.) If the condensing plate of the first jar had been directly connected with the earth, what would have been the potential of the collecting plate with the same charge?

Answer. 2·95.

(54.) A disc of tinfoil 10 centimetres in radius was pasted upon one face of a large sheet of glass. What amount of work will have to be expended in charging it with 100 units of electricity?

Since the plate is charged on one surface only, its capacity is $C = \frac{r}{\pi}$ and if $V =$ potential, $Q =$ charge, and $W =$ work required, we shall have $W = \frac{V}{2} \times Q = \frac{Q^2}{2C} = \frac{Q^2 \times \pi}{2r}$ $= \frac{\pi \times 10^3}{2} = 1570·8$ ergs.

(55.) If an exactly similar sheet of tinfoil had been pasted at the back of the glass and connected by a wire with the earth, find the amount of work required to impart to the collecting plate a charge of 100 units, the glass being one millimetre thick and its S.I.C. being 2.

In this case the capacity is $C = \frac{k\pi r^2}{4\pi t} = \frac{r^2}{2t}$;

$$\therefore W = \frac{Q^2}{2C} = \frac{Q^2 t}{r^2} = \frac{10^3}{10^2} = 10 \text{ ergs.}$$

(56.) If a second exactly similar and equal condenser had been placed behind the first in 'cascade,' what amount of work would have been required to impart a charge of 100 units to the collecting plate of the first?

Referring to Example 50 we have

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{2\,Q}{C};$$

$$\therefore W = \frac{V}{2} \times Q = \frac{Q^2}{C} = \frac{2\,Q^2\,t}{r^2} = 20 \text{ ergs.}$$

(57.) Had two more exactly equal and similar condensers been placed behind this second one, what amount of work would have been requisite to charge the collecting plate of the first with 100 electrical units?

Answer. 40 ergs.

(58.) What amount of work would be required to charge the collecting plate of A in Example 50 with 157 units?

Answer. 628 ergs.

(59.) A sphere of 5 centimetres radius is inclosed in a hollow concentric uninsulated sphere of 5·3 centimetres internal radius. Through a small hole in the latter a long wire passes to a distant insulated sphere of 5 centimetres radius. If a charge of 1000 units be communicated to the system, what will be the charge on each sphere?

Answer. Charge on enveloped sphere = 946·4 units.
„ „ second „ = 53·6 „

GUARD-RING ELECTROMETERS.

(1.) The area of the trap-door of a guard-ring electrometer was 16 square centimetres, and when the plates were not charged a force equal to the weight of 2 grammes was requisite in order to bring the trap-door down flush with the guard-ring. The movable plate was then connected with a conductor, the potential of whose charge was required, and the trap-door was again flush with the guard-ring when the plates were 5 millimetres apart. Taking $g = 981{\cdot}4$, find the potential of the conductor.

Let V_1 = potential of trap-door; V_2 = potential of movable plate, d = distance between the plates in centimetres, ρ = electric density on the movable plate; then

$$\rho = \frac{V_2 - V_1}{4\pi d}.$$

But the force per unit area urging the trap-door towards the other plate is $2\pi\rho^2$, and if A = area of trap-door in square centimetres, and F = force in dynes, which is required to bring it down flush with the guard-ring, then

$$F = 2\pi\rho^2 A = 2\pi A\left(\frac{V_2 - V_1}{4\pi d}\right)^2 = A\frac{(V_2 - V_1)^2}{8\pi d^2};$$

$$\therefore V_2 - V_1 = d\sqrt{\frac{8\pi F}{A}}.$$

In the present case $V_2 = V$ and $V_1 = 0$, $F = 2 \times 981{\cdot}4$; $A = 16$,

$$\begin{aligned} \therefore V &= {\cdot}5\sqrt{\frac{8\pi \times 2 \times 981{\cdot}4}{16}} \\ &= {\cdot}5\sqrt{\pi \times 981{\cdot}4} \\ &= 27{\cdot}76. \end{aligned}$$

(2.) A small uninsulated circular metal plate, whose area was 174 centimetres, was surrounded by a guard-ring, and was attached to one end of a delicate scale beam, at the other end of which was a counterpoise. Below this plate, and at a distance of 1·8 centimetre, was a larger circular metal plate, insulated and charged to the potential 17·5. Find the weight in grammes which would have to be added to the counterpoise so as to keep the upper plate flush with the guard-ring, taking $g = 981{\cdot}17$.

Let x = weight required in grammes, then $F = x \times 981{\cdot}17$ dynes; and from the solution of Example 1 we have

$$F = \frac{V^2 \times A}{8\pi d^2} = \frac{(17\cdot5)^2 \times 174}{8\pi \times (1\cdot8)^2} \text{ dynes,}$$

$$\therefore x = \frac{(17\cdot5)^2 \times 174}{8\pi \times (1\cdot8)^2 \times 981\cdot17} = \cdot667 \text{ grammes nearly.}$$

$$= 667 \text{ milligrammes.}$$

(3.) On another occasion, with the same instrument, the distance between the plates was 2·54 millimetres, and it was found that the counterpoise had to be increased by 965 milligrammes so as to keep the movable plate flush with the guard-ring. What was the potential of the lower plate? *Answer.* 2·97.

(4.) The area of the trap-door in a portable electrometer was 1·5 square centimetre, and for a variation of potential equal to one electrostatic unit the movable plate had to be shifted through a distance of ·508 millimetre. Express in milligrammes weight the force of torsion on the trap-door. *Answer.* 23·57 milligrammes.

(5.) In using this instrument for observations on the electrical condition of the atmosphere it was found that, on moving the instrument through a vertical distance of one foot, the movable plate had to be shifted through a distance of ·196 millimetre. What was the difference in electrical potential at these two points? *Answer.* ·385 unit.

(6.) The area of the trap-door of a 'long-range' electrometer was 10 square centimetres, and the torsional force on the trap-door spring was equal to the weight of 350 milligrammes. When the plates were connected to the opposite poles of a Holtz machine in work the distance between the plates was 6·6 centimetres when equilibrium was established. Find the difference of potential of the poles of the Holtz machine taking $g = 981\cdot17$. *Answer.* 193·9 units.

(7.) If in this instrument one turn of the screw which moves the lower plate corresponds to a motion of ·508 millimetre, and the screw head is divided into hundredths, find the difference of potential corresponding to a motion of one division of the screw head. *Answer.* ·0149 unit.

ELECTRICAL DISCHARGES.

(1.) The inner coating of a Leyden battery was connected with one of the balls of a Riess's spark micrometer, the other ball and the outer coating being in connection with the earth. When a charge of 700 units was given to the battery a spark passed between the balls of the micrometer when the movable one was brought to a distance of 2 millimetres from the other. What charge would have to be given to this battery so as to get a spark to pass at 2·5 millimetres?

For *small* variations of potential the discharging distance varies directly with the variation of potential.

Let $Q =$ charge of battery, $C =$ its capacity, $V =$ its potential.

Then, since $V = \frac{Q}{C}$ we have $\frac{V_2}{V_1} = \frac{Q_2}{Q_1}$ because C is constant,

and in the present case $Q_2 = Q_1 \times \frac{V_2}{V_1} = 700 \times \frac{2·5}{2}$

$= 875$ units.

(2.) A charge of 1030 units given to a Leyden battery of 3 equal jars gave a discharging distance of 4 millimetres. At what distance would the spark have passed if the same charge had been given to 4 jars exactly equal to the above? *Answer.* 3 millimetres.

(3.) If a battery of 4 equal jars, charged with 900 units, discharges at a distance of 1·3 millimetre, what will be the discharging distance between the balls of the micrometer when the same battery is charged with 1150 units?

Answer. 1·65 millimetre.

(4.) If the battery consisted of 3 equal jars like the last, and a charge of 1180 units were given to it, what would have been the sparking distance?

Answer. 2·28 millimetres, nearly.

(5.) If a battery of 5 Leyden jars, charged with 970 units, has a sparking distance of 1·5 millimetre when the barometer stands at 76 centimetres, what will be the sparking distance of a battery of 7 Leyden jars, each having twice the area of coated surface of those in the first battery, when charged with 1100 units, the barometer standing at 77 centimetres?

For a given charge the *sparking distance* varies inversely with the barometric pressure, and ∴ the distance required

$$= 1\cdot5 \times \frac{76}{77} \times \frac{110}{97} \times \frac{5}{14} = \cdot6 \text{ millimetre, nearly.}$$

(6.) A battery of 8 equal Leyden jars, charged with 100 units, has a sparking distance of 2 millimetres when the barometer stands at 748 millimetres. What will be the sparking distance for a battery of 11 jars exactly like the above when charged with 1300 units, the barometer standing at 752 millimetres? *Answer.* 1·88 millimetre.

(7.) A certain Leyden jar when charged with 50 units has a sparking distance of 2 millimetres. If another exactly equal Leyden jar be placed behind it in 'cascade' and then a charge of 50 units be imparted to the first, what will be the sparking distance between its knob and the outer coat of the second?

Referring to Example 50 in Statical Induction we find that the potential of the charge in the first jar will be twice as great as it was before, and therefore

Sparking distance $= 2 \times 2 = 4$ millimetres.

(8.) If a Leyden jar when charged with 100 units has a sparking distance of 1·5 millimetres, what charge will have to be given to it when it is placed as the first of a series of 4 exactly equal jars, so that its sparking distance may be 2 millimetres? *Answer.* $33\frac{1}{3}$ units.

(9.) A sphere of 4 centimetres diameter when charged to potential 18 gave a sparking distance of 1 millimetre, but when its potential was raised to 500 it discharged itself spontaneously into the air. Calculate the electrical pressure on the air and compare it with the atmospheric pressure.

Let r = radius of sphere. V = its potential and ρ its electrical surface density, then $\rho = \frac{r\text{V}}{4\pi r^2} = \frac{\text{V}}{4\pi r}$,

and the electrical pressure per unit area

$$= 2\pi\rho^2 = \frac{\text{V}^2}{8\pi r^2} = \frac{(500)^2}{8\pi \times 4} = 2486{\cdot}7 \text{ dynes.}$$

$$= \frac{2486{\cdot}7}{981{\cdot}17} = 2{\cdot}534 \text{ grammes weight.}$$

The atmospheric pressure may be taken as about equal to the weight of 1,014 grammes per square centimetre, and

$$\therefore \frac{\text{atmospheric pressure}}{\text{electrical pressure}} = \frac{1014}{2{\cdot}534} = \frac{400}{1}.$$

(10.) Two slightly convex parallel plates were charged so that the difference of their potentials was 4·26, and a spark passed when they were brought to a distance of ·019 centimetre. What was the electrical pressure from either surface just before the discharge took place?

The capacity of a condenser formed of two large plates of area A is $\text{C} = \frac{\text{A}}{4\pi t}$,

and $\therefore$ capacity per square centimetre $= \frac{1}{4\pi t}$.

But the density being the charge per unit area is given by

$$\rho = \frac{\text{V}}{4\pi t} = \frac{4{\cdot}26}{4\pi \times {\cdot}019};$$

$\therefore$ Electrical pressure per square centimetre

$$= 2\pi\rho^2 = 2\pi\left(\frac{4{\cdot}26}{4\pi \times {\cdot}019}\right)^2$$

$$= 1998{\cdot}3 \text{ dynes.}$$

$$= \frac{1998{\cdot}3}{981{\cdot}17} = 2{\cdot}037 \text{ grammes weight.}$$

(11.) If the difference of potential was 5·64, and the spark passed when the distance between the plates was ·0281 centimetre, what was the electrical pressure per square centimetre just before the discharge ?

Answer. 1602·9 dynes, or 1·6337 grammes weight.

(12.) If a spark passed between two parallel conducting surfaces when their distance was ·0408 centimetre, and their difference of potential 6·18, compare their electrical pressure just before discharge with the atmospheric pressure, which may be taken as 1013·8 grammes per square centimetre.

Answer. Air pressure=1089·6 times the electrical pressure.

(13.) In one of Wheatstone's experiments on the duration of the electric spark the mirror made 800 revolutions per second, and the distance of the spark from the mirror was 304·8 centimetres. The reflected image of the spark was then seen as an elongated streak of 24°. What was the duration of the spark ?

For an angular displacement of the mirror represented by θ the corresponding angular displacement of the reflected image of the spark is 2θ.

∴ while reflected image moved through 24° the mirror moved through 12°. But in one second the mirror moved 390 × 800 = 288000°.

∴ Time occupied in moving through 12° = duration of spark

$$= \frac{12}{288000} = \frac{1}{24000} \text{ second.}$$

(14.) In another experiment with the same mirror the elongated streak was 86° in angular measure. What would this give for the time of duration of the spark ?

Answer. ·00015 second.

(15.) Wheatstone connected the coatings of a Leyden jar with a conductor which at one part had in it three narrow gaps, the central gap being separated from each of the two side ones by a quarter of a mile of wire. When the jar was discharged and the reflected images of the three sparks were viewed in the revolving mirror as described above, the lateral displacement of the middle spark was $\frac{1}{2}$ a degree. What interval of time elapsed between the middle spark and the two side ones? *Answer.* $\frac{1}{1152000}$ of a second.

(16.) A Leyden jar charged with 145 units of electricity had a sparking distance of 2 millimetres. On recharging it, only 125 units of charge were required for a spark to pass at the same distance. What proportion did the *residual* charge bear to the entire charge?

Residual charge = 145–125 = 20 units;

$$\therefore \frac{\text{Residual charge}}{\text{Entire charge}} = \frac{20}{145} = \frac{1}{7}, \text{ nearly.}$$

(17.) The balls of a spark micrometer were connected to the coatings of a Leyden jar which was charged with 60 units, and discharge took place at a distance of 1·2 millimetre. To produce the next discharge at the same distance only 50 units of charge had to be given. What was the ratio of the residual to the entire charge? *Answer.* $\frac{1}{6}$.

(18.) A Leyden jar, the glass of which is 1·2 millimetre thick, and whose S.I.C. is 2, is coated with tinfoil to a height of 10 centimetres, and the inner radius of its base is 4 centimetres. It is charged to potential 8 and then completely discharged. Find the energy of the discharge.

$$\text{Capacity of jar} = k \times \frac{\pi r(r + 2h)}{4\pi t} = \frac{2 \times 4 \times 24}{4 \times {\cdot}12} = 400 \text{ units.}$$

$$\text{Charge} = \text{Q} = \text{C} \times \text{V} = 400 \times 8 = 3200.$$

But the work expended in charging

$$= \text{Q} \times \frac{\text{V}}{2} = 3{,}200 \times 4 = 12{,}800 \text{ ergs};$$

and since the jar is assumed to be completely discharged, the energy of the discharge must be equivalent to the work stored up in charging the jar.

$$\therefore \text{ Energy of discharge} = 12{,}800 \text{ ergs.}$$

(19.) A Leyden battery of 12 equal jars, the capacity of each of which was 1,000 units, was charged to potential 100. Express the energy of the complete discharge in thermal units. One water-gramme-degree centigrade of heat corresponding to $4{\cdot}2 \times 10^7$ ergs.

Answer. 6×10^7 ergs $= 1\frac{3}{7}$ w.g.d. C°.

(20.) A Leyden battery was formed of 12 equal jars. The coated area of each was 1000 square centimetres, and the thickness of the glass expressed by its equivalent of air was ·07 centimetre. Calculate the electrical energy of this battery when charged to potential 10.

Let n = number of jars ; A = area of coated surface of each jar.

t = *reduced* thickness of each jar ; V = potential of charge: then

$$\text{Capacity of battery} = \frac{n\,\text{A}}{4\,\pi\,t}\,; \quad \text{charge} = \frac{n\,\text{A}}{4\,\pi t} \times \text{V}.$$

Energy of battery = work expended in charging it

$$= \frac{n\,\text{A}}{4\,\pi\,t} \times \frac{\text{V}^2}{2} = \frac{12 \times 1000 \times 100}{8\,\pi \times {\cdot}07} = 682{,}091 \text{ ergs.}$$

(21.) If the 'reduced' thickness of glass had been ·15 centimetre, all else remaining as before, what would have been the electrical energy of the charge?

Answer. 318,309 ergs.

(22.) Two Leyden jars had exactly the same area of coated surface, and were charged to the same potential, but the glass in one was $1\frac{1}{2}$ times as thick as in the other. Compare the electrical energies of their charges.

Answer. $\dfrac{\text{Energy in thin jar}}{\text{Energy in thick jar}} = \dfrac{3}{2}.$

(23.) In the case of the jars mentioned in the last Example, suppose that the same quantity of electricity were imparted to each jar; compare the energy of the two charges. *Answer.* $\frac{\text{Energy in thick jar}}{\text{Energy in thin jar}} = \frac{3}{2}$.

N.B. The student should notice from these Examples that where the charge is constant the electrical energy of a Leyden battery varies directly with the thickness of the glass, and inversely with the area of coated surface, and ∴ a few small and thick ones would be better than many thin ones. If, on the other hand, as is generally the case in practice, the potential of the charge is constant, then the electrical charge varies directly with the area of coated surface, and inversely with the thickness of glass, and in this case it is advisable to use a number of small thin jars.

(24.) A Leyden jar, whose capacity was 1000 units, was charged to potential 10, and another jar, of capacity 500, was charged to potential 5. The outer coatings were then put to earth and the knobs connected together. Calculate the electrical energy of the system before and after the knobs are in contact.

Let c_1 = capacity of large jar, v_1 = potential of large jar,
c_2 = ,, small ,, v_2 = ,, small ,,
c = ,, the pair, v = joint potential after contact.

Then we have $c\,v = c_1 v_1 + c_2 v_2$, and $c = c_1 + c_2$;

$$\therefore \; v = \frac{c_1 v_1 + c_2 v_2}{c_1 + c_2}.$$

Electrical energy of pair *before* contact

$$= \frac{c_1 v_1^2 + c_2 v_2^2}{2} = \frac{1{,}000 \times 100 + 500 \times 25}{2} = 56{,}250 \text{ ergs.}$$

Electrical energy of pair *after* contact

$$= \frac{c\,v^2}{2} = \frac{(c_1 v_1 + c_2 v_2)^2}{2\,(c_1 + c_2)} = \frac{(100 \times 10 + 500 \times 5)^2}{2 \times 1500}$$
$$= 52{,}083 \text{ ergs.}$$

The difference between these two values, viz. 4,167 ergs, represents the amount of energy expended in the spark which passes between the knobs at the instant when they are connected.

(25.) Suppose that the smaller jar in the last example had no initial charge. Calculate the energy of the system before and after the knobs are in contact.

Answer. Before contact, 50,000 ergs. After contact, $33,333\frac{1}{3}$ ergs.

(26.) A Leyden battery, which had a coated surface of 5 square metres, and in which the thickness of the glass, reduced to its equivalent of air, was 0·7 millimetre, was charged by a Voss machine to potential 100. If the residual charge is $\frac{1}{9}$th of the whole, express in thermal units the energy which runs down in the discharge.

Answer. 6·015 w.g.d. C°.

(27.) A jar of 800 units capacity was charged to potential 60, and was then made to share its charge with two exactly similar but previously uncharged jars. What was the energy of the system before and after contact?

Answer. Before contact, $1{\cdot}44 \times 10^6$ ergs. After contact, $4{\cdot}8 \times 10^5$ ergs.

(28.) A Leyden jar of 1000 units capacity was charged to potential 8, and the charge was then shared with another equal jar which was previously uncharged. What proportion of the energy of the original charge runs down in the spark? *Answer.* One-half.

MOMENTS OF TORSION AND INERTIA.

(1.) A wire 25 centimetres long had attached to its lower end a vertical cylinder whose mass was 1000 grammes and radius 2·5 centimetres. This cylinder was then twisted and let go, and the mean of several observations gave 242″ as the time of 10 complete oscillations. What was the torsional moment of this wire for a twist of 1°?

Let f represent the force, expressed in dynes, which must be applied at right-angles to the end of a lever arm of one centimetre, so as to counterbalance the reaction due to the torsional moment of the wire when the cylindrical mass has been twisted through one radian,[1] then for any other angle of θ radians the moment of torsion will be

$$F = f.\,\theta.$$

Now the torsional reactive force depends entirely upon the molecular condition of the particular wire employed, and in no way upon the mass or the nature of the suspended cylinder. But inasmuch as it is this reactive force, due to torsion of the wire, which makes the cylinder oscillate, the rate of oscillation must depend upon the mass of the cylinder and upon its radius. Hence, by the laws which govern the motion of compound pendulums, we have

$$t = 2\pi\sqrt{\frac{K}{f}}; \quad \therefore f = \frac{4\pi^2 K}{t^2};$$

where K = moment of inertia of cylinder about the axis of rotation

$= m\dfrac{r^2}{2}$,

m = mass of cylinder in grammes,
r = its radius in centimetres,
t = time of one complete oscillation in seconds.

In the present case we have

$$\begin{aligned} f &= \frac{4\pi^2}{(24{\cdot}2)^2} \times 1000 \times \frac{(2{\cdot}5)^2}{2} \\ &= 210{\cdot}9 \text{ dynes}, \\ &= \frac{210{\cdot}9}{981{\cdot}17}, \text{ or } {\cdot}215 \text{ gravitation unit of force;} \end{aligned}$$

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[1] The radian is the angle subtended at the centre of a circle by an arc equal to the radius. It is an angle of $\frac{180^\circ}{\pi} = 57^\circ{\cdot}2958$, or about $57^\circ{\cdot}3$. The radian is the unit in which angular velocity is expressed.

which means that for a twist on the wire of one radian a force equal to the weight of ·215 of a gramme would have to be applied at the end of a lever arm of one centimetre, so as to balance the torsional reaction of the wire.

For an angle of 1° we shall have

$$F = f\frac{\pi}{180} = \frac{210{\cdot}9 \times \pi}{180} = 3{\cdot}68 \text{ dynes, approximately.}$$

(2.) Suppose that the wire in the last example had been 50 centimetres long and of twice the diameter, what would have been the value of F for a twist of 1°?

By Coulomb's investigations we know that for wires of the same material the value of F varies inversely as the length and directly as the fourth power of the diameter;

$$\therefore\ F = 3{\cdot}68 \times \frac{2^4}{2} = 29{\cdot}44 \text{ dynes.}$$

(3.) A cylindrical mass of 250 grammes, whose radius was 2·5 centimetres, was fastened to the lower end of a brass wire 25 centimetres long, and the mean time of 10 complete oscillations was 57 seconds. Find the moment of torsion for a twist of 1° expressed in gravitation measure, taking $g = 981{\cdot}17$. *Answer.* ·016887 gramme-centimetre.

(4.) What would have been the torsional moment for a twist of 1° if the wire had been 35 centimetres long?

Answer. ·012062.

(5.) A cylindrical mass of 100 grammes whose radius was 2 centimetres was attached to a wire, so that its axis was vertical, and when twisted the time of one complete oscillation was found to be 10 seconds. Express in dyne-centimetres the moment of the force required to produce a twist of 1°. *Answer.* 1·378 dyne-centimetre.

(6.) For another wire the same cylindrical mass was employed and the time of a complete oscillation was found to be 31 seconds. Find the moment of the torsional force required to produce a twist of 1°.

Answer. ·1434 dyne-centimetre.

(7.) Calculate the moment of torsion for a twist of 1° for one of Coulomb's silver wires from the following experiment :—

A copper wire 8·2 centimetres long, and whose mass was 6·1614 grammes, was suspended to it horizontally by its centre, and the time of 5 complete oscillations was found to be 143″.

In this case $K = m\frac{l^2}{12}$, $l = 8·2$, $m = 6·1614$, $t = 28·6$,

and for a twist of one radian we have, by the formula in Example (1),

$$f = \frac{4\pi^2 K}{t^2} = \frac{\pi^2}{(14·3)^2} \times \frac{6·1614 \times (8·2)^2}{12} = 1·67 \text{ dyne};$$

$\therefore$ torsional moment for 1° $= \frac{1·67 \times \pi}{180} = ·029$ of a dyne-centimetre.

N.B. The student can form a concrete idea as to the magnitude of a force of a dyne from the consideration that in London it is very nearly equal to the weight of 1·02 milligramme.

(8.) Find the moment of inertia of a rectangular prism of steel about a vertical axis through its centre of gravity, the dimensions of the prism being as follows :—length = 30 centimetres, breadth = 4 centimetres, thickness = ·6 centimetre, and the specific gravity of steel being 7·8.

Let x and y represent the lengths of two horizontal edges of a rectangular parallelopiped whose mass is m, then its moment of inertia about an axis through its centre parallel to the third edge, which is vertical, is

$$K = m \times \frac{x^2 + y^2}{12}.$$

In the present case $m = 30 \times 4 \times ·6 \times 7·8 = 561·6$ grms.,

$$x = 30, y = 4;$$

$$\therefore K = 561·6 \times \frac{30^2 + 4^2}{12} = 561·6 \times \frac{916}{12} = 42,869, \text{nearly};$$

which means that the resistance which the inertia of this bar opposes to the action of any force tending to accelerate or retard its rotation about the given axis is equivalent to that of a mass of 42,869 grammes placed at a distance of one centimetre from the axis of rotation.

The student should notice carefully what m really represents, because of the confusion existing in many text-books on this point. A gramme of matter is a perfectly determinate quantity of matter, and does not vary from place to place. The force represented by the *weight of a gramme*—i.e. the 'pull' exerted by the earth on a mass of one gramme, varies from place to place with the value of g. The inertia of a body depends upon its mass and not upon its weight, and therefore, in the expression for K, m always means *mass*.

(9.) If the length of the bar had been 20 centimetres, its breadth 3, and its thickness ·6 centimetre, what would have been its moment of inertia about the same axis?

Answer. 9570·6.

(10.) If the width of the bar in Example 8 had been one millimetre, all other dimensions remaining the same, what would have been its moment of inertia?

When the parallelopiped is very narrow, the general formula reduces itself to

$$K = m \times \frac{x^2}{12} = 14{\cdot}04 \times \frac{30^2}{12} = 1053.$$

(11.) What would have been the value of the moment of inertia of this bar if calculated from the formula in Example 8? *Answer.* 1053·011.

(12.) A knitting needle is 22 centimetres long, and its mass is 3·3 grammes. What is its moment of inertia about a vertical axis through its centre at right angles to its length?

When the thickness is very small compared with the length, we have

$$K = m \times \frac{l^2}{12} = 3{\cdot}3 \times \frac{(22)^2}{12} = 133{\cdot}1.$$

(13.) The mass of another needle was 9·4 grammes, and its length 22·5 centimetres; find its moment of inertia about the same axis. *Answer.* 396·6, nearly.

(14.) A steel cylinder 31 centimetres long and 2·5 centimetres in diameter was suspended horizontally. Find its moment of inertia, the mass of the bar being 1188·5 grammes.

It can be proved by an easy application of the Integral Calculus that the moment of inertia of such a cylinder, whose mass $= m$, its length $= l$, and its radius $= r$, is

$$\text{K} = m\left(\frac{l^2}{12}+\frac{r^2}{4}\right) = 1188{\cdot}5\left(\frac{31^2}{12}+\frac{1{\cdot}25^2}{4}\right)$$

$$= 95{,}643{\cdot}2, \text{ nearly.}$$

(15.) Find the moment of inertia about a line through its centre at right angles to its axis of a metal cylinder 20 centimetres long, 1·5 centimetre in diameter, and whose mass is 81 grammes. *Answer.* 2711·4, nearly.

(16.) A magnetised iron disc, 10 centimetres in diameter, whose mass was 565 grammes, was suspended by a thin wire from its centre. Find its moment of inertia.

In the case of a disc whose mass is m and radius r, the moment of inertia is

$$\text{K} = m\frac{r^2}{2}$$

$$= 565 \times \frac{25}{2} = 7062{\cdot}5.$$

(17.) Had the mass of the disc been only 200 grammes, and its radius 3 centimetres, what would have been its moment of inertia? *Answer.* 900.

(18.) A bar magnet, 30 centimetres long, was suspended by a fine wire from its centre and made 6 oscillations a minute under the influence of the earth's magnetic force. A brass ring whose mass was 1093·6 grammes, and its mean diameter 14 centimetres, was laid on the bar concentrically with the axis of rotation, and the bar then made 4 oscillations in a minute. Find the moment of inertia of the bar.

Let t = time of oscillation of magnet alone,
t' = „ „ „ after the addition of the ring,
k = moment of inertia of the brass ring;

then it can easily be proved that

$$\text{K} = k \times \frac{t^2}{t'^2 - t^2} = \frac{1093{\cdot}6 \times 4900}{125}$$
$$= 42{,}869.$$

(19.) In the case of another bar magnet the number of oscillations in a minute was 10, and when a brass ring whose mass was 332·3 grammes, and diameter 12 centimetres, was placed on it, as in the last Example, it made 10 oscillations in a minute and a half. What was its moment of inertia? *Answer.* 9570·3.

(20.) A bar magnet, 20 centimetres long, made 10 oscillations in 95 seconds under the influence of the earth's magnetic force. Two cylindrical weights of 100 grammes each and whose diameters were each 2 centimetres were hung over the ends of the magnet by a thread, and the time of 10 oscillations was then 135 seconds. What was the moment of inertia of this magnet?

The moment of inertia of the added masses about the vertical axis through the centre of the magnet is

$$k = 200\left(10^2 + \frac{r^2}{2}\right) = 200 \times \frac{201}{2}$$
$$= 20{,}100;$$

and as before

$$\text{K} = k\frac{t^2}{t'^2 - t^2} = 20{,}100\frac{(9{\cdot}5)^2}{(13{\cdot}5)^2 - (9{\cdot}5)^2}$$
$$= 19{,}717{\cdot}6.$$

(21.) Had these two weights been hung over the ends of the magnet in Example 18, what would have been its time of oscillation? *Answer.* 14·33 seconds, nearly.

(22.) A bar magnet, suspended by its centre, made 25 oscillations in a minute under the action of the earth's magnetic force, and when two cylindrical masses of 20 grammes each and one centimetre diameter were attached at distances of 7 centimetres on each side of the axis of suspension, it made 20 oscillations in a minute. Find its moment of inertia. *Answer.* 3493·3.

(23.) A bar magnet, 12 centimetres long and 1·5 centimetre broad, whose mass was 135 grammes, was suspended by a thread without torsion and under the influence of the earth's magnetic force, made 50 complete oscillations in 7 minutes. Find the moment of the force which, if applied at a point whose distance from the axis of suspension was one centimetre, would be equivalent to the resultant action of the earth's magnetic force.

It can easily be proved by the elementary laws of Rigid Dynamics that if K be the moment of inertia of a compound pendulum and f the force expressed in dynes, which, if applied at the unit distance from the axis of oscillation, would be equivalent to the resultant action of the accelerating forces on the pendulum, then, as in Example 1,

$$t = 2\pi\sqrt{\frac{K}{f}}; \text{ whence } f = \frac{4\pi^2}{t^2} \times K.$$

In the present case $t = 8·4$ seconds;

$$K = 135 \times \frac{12^2 + (1·5)^2}{12}$$

$$= \frac{135 \times 146·25}{12}.$$

$$\therefore f = \frac{4\pi^2}{(8·4)^2} \times \frac{135 \times 146·25}{12}$$

$$= 920·5 \text{ dynes.}$$

(24.) Calculate the value of f for the bar magnet described in Example 18. *Answer.* 16,890 dynes.

MAGNETISM.

(1.) What is the force with which a magnetic pole whose strength is 20 units repels a similar magnetic pole of 15 units strength at a distance of 10 centimetres?

The unit magnetic pole is one of such a strength that it repels an equal pole placed at one centimetre distance with a force of one dyne.

Let f represent the force between two magnetic poles whose strength are m_1 and m_2 and their distance d, then we have

$$f = \frac{m_1 \times m_2}{d^2}$$

$$= \frac{20 \times 15}{100} = 3 \text{ dynes.}$$

(2.) A magnetic pole of 90 units strength is placed at a distance of 15 centimetres from a similar pole of 60 units strength; find the force between them.

Answer. 24 dynes.

(3.) A magnetic pole of strength 5 is placed in a magnetic field of strength ·48; what will be the force experienced by this pole?

The intensity of a magnetic field is measured by the force which a unit pole will experience when placed in it. The force exerted on a pole of strength m when placed in a field of strength I is

$$f = m \times \text{I}.$$

In the present case $f = 5 \times \cdot 48 = 2 \cdot 4$ dynes.

N.B. The field of unit intensity is that which exerts a force of one dyne upon a unit pole.

(4.) What is the strength of a magnetic pole which experiences a force of 10 dynes when placed in a magnetic field whose intensity is ·7898? *Answer.* 12·661.

(5.) A bar magnet whose strength is 128, and the distance between its poles 10 centimetres, is placed in a field of intensity 2·2 in a direction at right angles to the lines of magnetic force. Find the strength of the *couple* experienced by the magnet.

Let G represent the strength of the couple, I the intensity of the field, m the strength of each pole, and l the distance between them; then

$$\text{G} = ml \times \text{I} = 128 \times 10 \times 2{\cdot}2 = 2816 \text{ dynes.}$$

N.B. The product of the strength of one of the poles of a magnet by the distance between the two poles, or ml, is called the *magnetic moment* of the magnet, and the student will notice that when this is multiplied by the intensity of the field the product gives the numerical value of the couple acting upon the magnet when it is placed in the field with its axis at right angles to the lines of force.

(6.) Find the magnetic moment of a bar magnet, which when placed in a uniform field of force of strength ·18, and in a direction at right angles to the lines of force, is twisted with a force equal to the weight of one gramme, taking $g = 981{\cdot}17$. *Answer.* 5451, nearly.

(7.) The north pole of a very long vertical magnet whose strength was 250 was placed at a perpendicular distance of 20 centimetres from the centre of a horizontal magnetic needle whose length was 5 centimetres and strength 50. Find the moment of the couple acting upon the small magnet.

In Fig. 3 let $ns = 2l$, $n\text{N} = r$, $\text{ON} = d$, $\text{O}n\text{N} = \theta$, $\text{P} =$ strength of pole N, and $p =$ strength of pole n or s.

Then the forces acting upon ns are a force $\dfrac{p \times \text{P}}{(\text{N}\,n)^2}$ in the direction N n and an equal force $\dfrac{p \times \text{P}}{(\text{N}\,s)^2}$ in the direction s N, and ∴ the strength of the couple which tends to twist the small magnet about its centre O is given by the expression,

$$\text{G} = 2l\frac{p \times \text{P}}{r^2} \times \sin\theta = 2l\frac{p \times \text{P}}{r^2} \times \frac{d}{r}$$

$$= \frac{p \times \text{P} \times 2l \times d}{(d^2 + l^2)^{\frac{3}{2}}}$$

$$= \frac{50 \times 250 \times 5 \times 20}{(20^2 + 2{\cdot}5^2)^{\frac{3}{2}}} = \frac{50 \times 250 \times 100}{(406{\cdot}25)^{\frac{3}{2}}} = 152{\cdot}658.$$

(8.) The same magnet was then placed at a perpendicular distance of 10 centimetres from the centre of the small magnet. Find the strength of the couple acting upon it.

Answer. 570·672.

(9.) If the strength of the pole of the long magnet had been 100, its distance from the small magnet 50 centimetres, the length of the small magnet 10 centimetres, and its strength 50, what would have been the moment of the couple? *Answer.* 19·7, nearly.

(10.) A bar of soft steel, whose length is 30 centimetres and its sectional area 1·5 square centimetre, is placed in a uniform magnetic field of strength ·18. The coefficient of magnetic induction being 21·6, find its magnetic moment.

If I = intensity of magnetic field, k = coefficient of magnetic induction, l = length of bar, and s = area of its cross-section, then the magnetic moment of the bar is

$$sl(k\,\text{I}) = 1{\cdot}5 \times 30 \times 21{\cdot}6 \times {\cdot}18 = 174{\cdot}96.$$

(11.) What is the intensity of magnetisation of this piece of steel?

$$\text{Intensity of magnetisation} = \frac{\text{magnetic moment}}{\text{volume}}$$

$$= \frac{174{\cdot}96}{1{\cdot}5 \times 30} = 3{\cdot}888.$$

(12.) In order to magnetise a steel bar it was placed in a uniform magnetic field whose strength was 30. The magnet was 30 centimetres long and 1·75 square centimetres in sectional area, and the coefficient of magnetic induction was 17·4. What was its magnetic moment and the intensity of magnetisation? *Answer.* 27,405 and 522.

(13.) The magnetic moment of a steel bar magnet weighing 257 grammes is 3206 units; assuming its specific gravity to be 7·8, what is its intensity of magnetisation? *Answer.* 97·3.

(14.) A steel bar weighing 400 grammes was placed in a magnetic field whose strength was 23, and the magnetic moment was afterwards found to be 18,990 units. The specific gravity of the steel being 7·8, what was the coefficient of magnetic induction?

By Example 10 we have

$$\kappa = \frac{\text{magnetic moment}}{\text{I} \times \text{vol. of bar}} = 16{\cdot}1.$$

(15.) Two magnetic needles, which are exactly alike in all respects except in being unequally magnetised, are freely suspended in the same uniform magnetic field, and are observed to make 12 and 17 oscillations in a minute respectively. Compare their magnetic intensities.

Since all the laws of pendular motion are applicable to the case of a magnet which is oscillating under the influence of the earth's magnetic force, it follows that the magnetic force which causes a magnet to oscillate varies inversely as the square of the time of one oscillation.

If, therefore, I_1 and I_2 represent the intensities of the magnets, and t_1 t_2 their times of oscillation, we have

$$\frac{I_1}{I_2} = \left(\frac{t_2}{t_1}\right)^2 = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{12}{17}\right)^2 = \frac{144}{289} = \tfrac{1}{2}, \text{ nearly.}$$

(16.) If two magnetic needles, when oscillating in the same magnetic field, made 375 and 242 oscillations respectively in 10 minutes, compare the intensities of the magnetic forces. *Answer.* As 2 : 1, nearly.

(17.) A small bar magnet was placed in a paper stirrup and suspended by a thread of unspun silk, and was observed to make 9 oscillations in a minute. It was then removed, remagnetised, replaced, and then the number of oscillations per minute was found to be 18. In what ratio had its magnetism been increased? *Answer.* As 4 : 1, nearly.

(18.) A small magnetic needle, when suspended horizontally by a thread of unspun silk, makes 100 oscillations in 7 minutes 20 seconds, under the influence of the earth's magnetic force. When the south pole of a long magnet A is placed at a certain distance north of it, it makes 100 oscillations in 5 minutes 6 seconds, and when the south pole of another magnet B replaces the pole A, the time of 100 oscillations is 4 minutes 28 seconds. Compare the strengths of the magnet poles of A and B.

Let $f =$ force causing the magnet to oscillate when only under the earth's influence;
$f_1 =$ force causing the magnet to oscillate when under influence of earth + magnet A;
$f_2 =$ force causing the magnet to oscillate when under influence of earth + magnet B;
t, t_1, t_2 the respective times of one oscillation.

Then we have

$$\frac{f_1}{f} = \left(\frac{t}{t_1}\right)^2, \quad \therefore \frac{f_1 - f}{f} = \frac{t^2 - t_1^2}{t_1^2}; \text{ similarly } \frac{f_2 - f}{f} = \frac{t^2 - t_2^2}{t_2^2}.$$

$$\therefore \frac{\text{strength of A}}{\text{strength of B}} = \frac{f_1 - f}{f_2 - f} = \frac{t^2 - t_1^2}{t^2 - t_2^2} \times \left(\frac{t_2}{t_1}\right)^2$$

$$= \frac{(t - t_1)(t + t_1)}{(t - t_2)(t + t_2)} \times \left(\frac{t_2}{t_1}\right)^2$$

$$= \frac{1{\cdot}34 \times 7{\cdot}46}{1{\cdot}72 \times 7{\cdot}08} \times \left(\frac{2{\cdot}68}{3{\cdot}06}\right)^2 = {\cdot}63, \text{ nearly.}$$

(19.) A magnetic needle makes 12 oscillations a minute under the influence of the earth alone. Under the combined influence of the earth and one pole of a magnet A it makes 14 oscillations, and when the magnet A is replaced by another magnet B it makes 16 oscillations in a minute. Compare the strengths of the two magnets A and B.

Answer. As 13 : 28.

(20.) Under the influence of the earth's magnetic force a small magnet when freely suspended made 33 oscillations in 2 minutes. The north pole of a very long magnet was then placed at a distance of 3 centimetres from the south pole of the suspended magnet, and the number of oscillations per minute was then 38. On removing this north pole to a distance of 6 centimetres from the small magnet, the latter made 23 oscillations in a minute. Compare the forces exerted by the bar magnet upon the suspended one at these two distances. *Answer.* As 456 : 100, nearly.

(21.) A small magnetic needle when under the influence of only the earth's magnetic force makes 20 oscillations in 100 seconds. One pole of a long bar magnet (A) is then brought near the needle, which is observed to make 20 oscillations in 25 seconds. Compare the strength of this magnetic pole with the horizontal intensity of the earth's magnetic force.

Let H represent the horizontal intensity of the earth's magnetic force;
P represent the strength of the magnet pole.

Then, since in the second case 80 oscillations are performed in 100 seconds,

$$\frac{H}{H+P} = \left(\frac{20}{80}\right)^2 = \frac{1}{16}; \; \therefore P = 15\,H.$$

(22.) Another magnet B was then placed in the position previously occupied by A, and the needle now made 20 oscillations in 15 seconds. Compare the strengths of the poles of A and B. *Answer.* P : P′ = 135 : 391.

(23.) A dipping needle makes 25 oscillations in a minute in the magnetic meridian at one place, and when taken to another place it makes 28 oscillations in the same time.

Compare the intensities of the earth's magnetic field at the two places. *Answer.* As 625 : 784.

N.B. The results obtained by using a dipping needle in this manner are liable to several kinds of errors, and it is usual to determine the values of the horizontal intensity (H) and of the angle of dip (i) by independent methods, and then to calculate the value of the total intensity (I) from the formula

$$I = \frac{H}{\cos i}.$$

(24.) A magnetic needle is suspended horizontally in the magnetic meridian, and is then drawn away from the meridian (1) through a right angle and (2) through an angle of 45°. Compare the forces which, in these two positions, tend to bring the needle back into the magnetic meridian.

Let N*s* be the line of the magnetic meridian, *n*O*s* the needle, making an angle θ with the meridian.

Let F represent the force acting upon the needle in a direction parallel to the meridian for any particular value of θ.

The resolved part of this force at right angles to the needle, which is the force tending to bring the needle back into the meridian, is $F \sin \theta$.

Hence if F_1 and F_2 be the forces for the two given positions,

$$\frac{F_1}{F_2} = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin 90^\circ}{\sin 45^\circ} = \frac{\sqrt{2}}{1} = \frac{1{\cdot}414}{1},$$

(25.) If the earth's magnetism exerts a force of 50 dynes to bring back into the magnetic meridian a horizontal bar magnet which has been displaced through a right angle, what will be the magnitude of the corresponding forces when the displacements are 20°, 40°, 60°, and 80°, respectively? *Answer.* 17·1 ; 32·14 ; 43·3 ; 49·24 dynes.

(26.) At one place on the earth's surface a declination needle makes 75, and at another place 80, oscillations in a minute. Compare the forces which tend to bring the needle back into the magnetic meridian.

Let f_1 and f_2 be the forces; then, as in Example 15, we have

$$\frac{f_1}{f_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{75}{80}\right)^2 = \frac{225}{256}.$$

(27.) A bar magnet when freely suspended is observed to make 20 oscillations in 5 minutes at Vienna, and, without having its magnetism at all altered, it is taken to Panama, and there makes 27 oscillations in 5 minutes. Compare the intensities of the horizontal components of the earth's magnetism at Vienna and Panama.

Answer. As 100 : 182, nearly.

(28.) A bar magnet whose magnetic moment was 2000 C.G.S. units was suspended horizontally in the magnetic meridian by the wire mentioned in Example 1 of the last Chapter. The horizontal intensity of the earth's magnetic field being ·18, find the angle through which the torsion head would have to be turned so as to draw the magnet 10° away from the meridian.

Let x° = required torsion of head piece.
y° = deflection of the magnet from the meridian.

The moment of the couple, due to the magnetic action of the field, which tends to bring the magnet back into the meridian is

$$= 2000 \times \cdot 18 \times \sin y^\circ \text{ dynes.}$$

The moment of the couple, due to the torsion of the wire, which tends to twist the magnet away from the meridian is

$$= (x - y)\ 3{\cdot}68 \text{ dynes};$$

and since these two couples must balance each other,

$$x - y = \frac{360 \times \sin y}{3{\cdot}68} = 16{\cdot}98 \text{ if } y = 10^\circ;$$

$$\therefore x = 26^\circ{\cdot}98 = 27^\circ, \text{ nearly.}$$

(29.) A wire of the same material and diameter as in the last Example, but of 3 times the length, was used to suspend another magnet horizontally in a magnetic field whose horizontal intensity was ·17. The magnetic moment of this bar being 3500 units, find the angle through which the torsion head would have to be turned, so as to make the angle between the magnet and the meridian 5°.

Answer. 47°·25.

(30.) A bar magnet was suspended by a wire in a torsion balance, so that when the axis of the magnet was in the magnetic meridian there was no twist on the wire. The torsion head was then turned through 8 right angles, and the magnet came to rest at an angle of 22° with the meridian. The torsional force of the wire for a twist of 1° had previously been found to be 2 dynes, and the earth's horizontal intensity was ·18. Find the magnetic moment of this bar magnet.

Let M = magnetic moment of this magnet,
H = earth's horizontal intensity,
θ = angular deflection of magnet from the meridian.
ϕ = angle through which the wire was twisted.
f = torsion coefficient of the wire for a twist of 1°.

Then $\phi = 2 \times 360 - \theta = 720 - 22 = 698°$.

The earth's magnetic force tends to bring the magnet back into the meridian with a force whose moment is $HM \sin\theta$, while the torsional force of the wire tends to twist it away from the meridian with a force represented by $f \times \phi$, and these two forces must balance each other. Hence

$$HM \sin\theta = f \times \phi\,;$$

$$\therefore\ M = \frac{f\phi}{H\sin\theta} = \frac{2 \times 698}{\cdot 18 \times \sin 22°} = 20{,}703\cdot 2.$$

(31.) Another magnet was suspended by a wire in the manner just described, and its deflection from the magnetic meridian was 30° when the torsion head was twisted through

360°. The torsion coefficient of the wire for 1° being 1·5 dyne, and the horizontal intensity of the earth's magnetic force being ·18, find the magnetic moment of this magnet.

Answer. 2750.

(32.) A bar magnet 10 centimetres long, 2 centimetres broad, and whose mass was 150 grammes, was suspended freely by its centre, and was observed to make 6 oscillations in a minute. Find the numerical value of the moment of the couple due to the earth's magnetic force.

Let K = moment of inertia of the bar,
G = strength of the couple acting on the bar, in dynes,
t = time of oscillation in seconds.

Then, as in Example 1 of the last Chapter,

$$G = \frac{4\pi^2}{t^2} \times K, \text{ and } K = 150 \times \frac{10^2 + 2^2}{12} = 1300,$$

$$\text{and } \therefore G = \frac{4\pi^2 \times 1300}{100} = 513{\cdot}22, \text{ nearly};$$

which means that the couple is equivalent to a force of 513·22 dynes acting at the end of a lever arm of one centimetre.

(33.) What was the magnetic moment of this magnet if the horizontal intensity of the earth's magnetic field was ·18? *Answer.* 2851°·2.

(34.) A small bar magnet was suspended in a torsion balance so that when there was no torsion on the wire the magnet was in the magnetic meridian. The torsion head was then turned through 80°, which produced a deflection of the magnet of 5°. The bar was then removed, re magnetised, and replaced. The torsion head was then turned through 230°, in order to deflect the magnet 5° from the meridian. In what ratio had the magnetism of the bar been increased? *Answer.* Three-fold.

(35.) Coulomb suspended a long thin bar magnet in a torsion balance so that when there was no twist on the wire the bar was in the magnetic meridian. He then turned the torsion head through 8 right angles, when the deflection of the magnet from the meridian was 20°. He then brought the torsion head back to zero, and placed near the north pole of the suspended magnet the like pole of a long magnet. The magnet was repelled, and in order to bring the deflection down to 17° and 12° the torsion head had to be twisted through 12 and 32 right angles respectively. Prove that the force of repulsion between the two north poles varied very nearly as the inverse square of the distance.

For angles up to 24° the sines of the angles are proportional to those angles as far as the third place of decimals.

From the first part of the experiment we find that for a deflection of 20° the directive action of the earth on the magnet is $2 \times 360 - 20 = 700^\circ$ of torsion, and ∴ for any other angle of x° it will be $\frac{700}{20}x = 35\,x^\circ$.

From the subsequent stages of the experiment we then find that

Angular separation of poles.	Force on magnet in degrees of torsion.
24° . . .	$24 + 24 \times 35 = 864^\circ$
17°	$3 \times 360 + 17 + 17 \times 35 = 1692^\circ$
12° . . .	$8 \times 360 + 12 + 12 \times 35 = 3312^\circ$,

∴ when the arcs were as 12 : 17 : 24 the forces were as 3312 : 1692 : 864. If the law of the inverse square had been exactly followed, the forces would have been as 3312 : 1650 : 828.

N.B. The magnets used by Coulomb in these experiments were 65 centimetres long and from 1 to 2·5 millimetres in diameter. In fact they were like long thin wire knitting needles.

(36.) N S and N′ S′ are two small bar magnets, whose axes are in the same straight line O O′, which is at right angles to the magnetic meridian ; *n s* is a very small magnet free to turn about its centre *o*, which is in the straight line O O′. If the distances O*o* and *o*O′ be 49 and 45 centimetres respectively, and the magnet *n s* shows no deflection, compare the magnetic moments of N S and N¹ S¹.

Let M = magnetic moment of N S (see Fig. 4),
$m =$,, ,, *n s*
$p =$ strength of each pole of the magnet N S,
S N $= 2l$,
O *o* $= d$.

The intensity of the magnetic field at *o* due to the pole N

$$=\frac{p}{(d-l)^2}.$$

The intensity of the magnetic field at *o* due to the pole S

$$=\frac{-p}{(d+l)^2}.$$

Since the distance N *o* is very much greater than the length of *n s*, the lines of force at *n* and *s* due to the poles N and S will be sensibly parallel and at right angles to the magnet axis of *n s*. Hence the intensity of the magnetic field at *o* due to the magnet S N is

$$=p\left\{\frac{1}{(d-l)^2}-\frac{1}{(d+l)^2}\right\}$$

$$=\frac{p}{d^2}\left\{\left(1-\frac{l}{d}\right)^{-2}-\left(1+\frac{l}{d}\right)^{-2}\right\}$$

$$=\frac{p}{d^2}\left\{1+\frac{2l}{d}+3\frac{l^2}{d^2}+\cdots-\left(1-2\frac{l}{d}+\frac{3l^2}{d^2}\cdots\right)\right\}$$

$$=\frac{p}{d^2}\left\{\frac{4l}{d}+\frac{8l^3}{d^3}+\cdots\right\}$$

$$=\frac{2p\times 2l}{d^3}\left\{1+\frac{2l^2}{d^2}+\cdots\right\}$$

$=\frac{2\text{M}}{d^3}$ if, as is generally the case, we may neglect all the terms of this series after the first.

Hence the magnetic moment of the couple due to the magnet N S, which tends to turn the magnet *n s* away from the meridian, is

$$G = \frac{2\,M\,m}{d^3}.$$

For the magnet N′ S′, which is placed in a similar position on the opposite side of *n s*, we shall have

$$G' = \frac{2\,M'\,m}{d'^3};$$

and if these two couples balance each other we must have

$$G = G';$$

$$\therefore \frac{M}{d^3} = \frac{M'}{d'^3};$$

$$\therefore \frac{M}{M'} = \left(\frac{d}{d'}\right)^3 = \left(\frac{49}{45}\right)^3 = \frac{129}{100}, \text{ nearly.}$$

(37.) Two small bar magnets, whose magnetic moments were as 7 : 3, were placed in the manner described in the last Example on opposite sides of a small magnetic needle *n s*. If the distance of the weaker magnet from the needle was 40 centimetres, what must have been the distance of the stronger one so as to balance the other?

Answer. 53 centimetres nearly.

(38.) A graduated metre scale was placed at right angles to the magnetic meridian, and a small compass needle was placed at its centre. A short bar magnet was placed due west of the needle at right angles to the meridian, and so that the distance between its centre and that of the compass needle was 45 centimetres. The deflection of the compass needle from the meridian was observed to be 12°·5. The horizontal intensity of the earth's magnetic force being ·18, find the magnetic moment of this bar magnet.

Let M = magnetic moment of N S (see Fig. 5),
m = " " $n\,s$,
H = horizontal component of earth's magnetic force,
θ = angle needle makes with meridian,
d = distance O o.

If the needle $n\,s$ were *in* the magnetic meridian, the moment of the couple tending to twist it out of the meridian, which it would experience owing to the presence of N S, would, by Example 36, be $\frac{2\,\mathrm{M}\,m}{d^3}$, and, since the needle makes an angle θ with the meridian, the moment of the couple is $\frac{2\,\mathrm{M}\,m}{d^3}\cos\theta$.

The couple due to the earth's horizontal component, which tends to twist the needle back into the magnetic meridian, is

$$\mathrm{H}\,m\sin\theta,$$

and when the needle is at rest these two couples must balance each other, and in that case

$$\frac{2\,\mathrm{M}\,m}{d^3}\cos\theta = \mathrm{H}\,m\sin\theta\,;$$

$$\therefore\ \mathrm{M} = \frac{\mathrm{H}\tan\theta}{2}\times d^3$$

$$= \frac{\cdot 18\times\tan 12^\circ\cdot 5\times 45^3}{2}$$

$$= 1818 \text{ nearly.}$$

(39.) If the magnetic moment of the magnet N S were 718, the distance O o were 45 centimetres, and the deflection of the compass needle 5°, what would have been the intensity of the horizontal component of the earth's magnetic force at the time and place of observation? *Answer.* ·18.

(40.) The bar magnet which Weber used to determine the absolute intensity of the earth's magnetic force was 10·1 centimetres long, 1·75 centimetres broad, and weighed 142 grammes. When suspended freely by its centre the mean time of a complete oscillation was found to be 13·34 seconds. He then placed a metre scale in a horizontal position at right angles to the magnetic meridian, with a small compass needle at its centre, and the above magnet on this scale so that it pointed east and west, and he found that when the distance between the centres of the magnets was 45 centimetres the deflection of the compass needle was 11° 24′, and when the distance was 35 centimetres the deflection was 23° 29′. Calculate from these data the value of the horizontal intensity of the earth's magnetic force.

The first part of the problem gives us the value of H M —that is, the moment of the couple with which the earth's magnetic force would act upon the bar magnet if it were placed horizontally at right angles to the meridian, and by the method employed in Example 32,

$$\text{H M} = \frac{4\pi^2 \text{K}}{t^2} = \frac{4\pi^2}{(13{\cdot}34)^2} \times 142 \times \frac{(10{\cdot}1)^2 + (1{\cdot}75)^2}{12}$$
$$= 275{\cdot}8 \text{ dyne-centimetres} \quad . \quad . \quad . \quad . \quad (1).$$

Let θ = angle of deflection of needle from the meridian.
d = distance between the centres of the two magnets.
M = magnetic moment of the bar magnet.

Then, by the method adopted in the solution of Example 38, we should have

$$\tan\theta = \frac{\text{M}}{\text{H}}\left\{\frac{2}{d^3} + \frac{4\,l^2}{d^5} + \cdots\right\}.$$

In the present case the length of the bar magnet as compared with the distance between the centres of the two magnets is so great that we must retain the second term of the series within the bracket, and we may then write

$$\tan\theta = \frac{x}{d^3} + \frac{y}{d^5} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

and determine x and y from the data of the experiment.

Multiplying equation (2) by d^3, we get

$$d^3 \tan\theta = x + \frac{y}{d^2}.$$

Let θ_1, d_1 and θ_2, d_2 be simultaneous values of θ and d, then, eliminating y between the two resulting equations, we get

$$x = \frac{d_2^5 \tan \theta_2 - d_1^5 \tan \theta_1}{d_2^2 - d_1^2} ;$$

$$\therefore \frac{2\text{M}}{\text{H}} = x = \frac{45^5 \tan 11^\circ 24' - 35^5 \tan 23^\circ 29'}{45^2 - 35^2}$$

$$= \frac{37207390 - 22818980}{800} = \frac{14388410}{800} \quad . \quad . \quad . \quad (3).$$

Dividing equation (1) by (3) we get rid of M, and obtain

$$\text{H}^2 = \frac{2 \times 275{\cdot}8 \times 800}{14388410}, \text{ whence } \text{H} = {\cdot}175, \text{ nearly.}$$

The number ·175 means that if a magnet of unit moment were placed horizontally at right angles to the magnetic meridian, the earth's magnetic force would act upon it with a couple whose moment is equal to that of ·175 dyne acting at the end of a lever arm of 1 centimetre.

(41.) If a third observation had given a deflection of 35° 18′ for a distance of 30 centimetres, and we combine this with the first of the observations in the last Example, what value should we have obtained for H ?

Answer. ·176, nearly.

(42.) A bar magnet 10 centimetres long and 1·5 centimetre broad, whose mass was 168·5 grammes, was freely suspended, and found to make 61 complete oscillations in 20 minutes. A metre scale with a small compass at its centre was placed horizontally at right angles to the magnetic meridian, and the above magnet was placed upon it as in Weber's experiment. When the distances between the centres of the two magnets were 35 and 30 centimetres the corresponding deflections of the compass needle were 10°·6 and 16°·4 respectively. What was the value of H ?

Answer. ·188.

(43.) In an experiment made by Gauss, the mean time of a complete oscillation of a certain bar magnet was 15·235 seconds, and its moment of inertia 43287·32. This magnet was placed in the manner described in the last Example, and two sets of observations were taken of the angular deflection of the compass needle and of the corresponding distance in centimetres between the centres of the compass needle and the bar magnet, with the following results :—

d	θ
120 centimetres	3° 42′ 19″
160 „	1° 34′ 19″

Find from these data the numerical value of the horizontal component of the earth's magnetic force at the time and place of observation. *Answer.* ·1804.

(44.) A bar magnet N S (see Fig. 6), whose length was 10 centimetres and its magnetic moment 1179, was placed horizontally at right angles to the magnetic meridian, so that its centre was in the same meridian with, and at a distance of 50 centimetres from, the centre of a small compass needle, which was deflected, by its action, through an angle of 3° from the meridian. Calculate from this the value of the horizontal component of the earth's magnetic force.

Let M = magnetic moment of N S
$m =$ „ „ $n\,s$
$p =$ strength of each pole of N S
$\text{N S} = 2l$
$\text{O}o = d$
$\theta =$ angle N oO

The strength of the magnetic field at o due to N $= \dfrac{p}{l^2+d^2}$

„ „ „ S $= \dfrac{-p}{l^2+d^2}$

The lines of force at o, due to the combined action of N and S, are parallel to the axis of N S, and therefore the resultant force at o is

$$\text{F} = \frac{2p}{l^2+d^2}\sin\theta = \frac{2pl}{(l^2+d^2)^{\frac{3}{2}}} = \frac{\text{M}}{(l^2+d^2)^{\frac{3}{2}}}.$$

Let a represent the angle which the compass needle makes with the meridian; then the moment of the couple due to N S, which tends to twist the needle away from the meridian, is

$$\frac{\mathrm{M}\,m\cos a}{(l^2+d^2)^{\frac{3}{2}}}.$$

The force with which the horizontal component of the earth's magnetism tends to twist the needle back into the meridian is $\mathrm{H}\,m\sin a$, and, since there is equilibrium, we must have

$$\mathrm{H}\,m\sin a = \frac{\mathrm{M}\,m\cos a}{(l^2+d^2)^{\frac{3}{2}}}, \text{ whence } \mathrm{H} = \frac{\mathrm{M}}{d^3}\cot a\left(1+\frac{l^2}{d^2}\right)^{-\frac{3}{2}}.$$

In the present case $\mathrm{H} = \frac{1179}{(50)^3}\cot 3^\circ = \cdot 18$ approximately.

(45.) If another bar magnet were placed in a similar position to N S, but at a distance of 45 centimetres from the centre of the compass needle, and produced a deflection of 4° 20′, calculate the magnetic moment of this bar magnet, the horizontal intensity of the earth's magnetic force being ·18. *Answer.* 1243, nearly.

(46.) A small magnet *n s* was suspended freely by its centre *o*, so as to oscillate in a horizontal plane. In this same plane was placed a second and larger bar magnet N S, perpendicular to the magnetic meridian, and so that its centre was in the same meridian with *o*. When the distance O *o* was 300 centimetres, the deflection of *n s* was 5′ 3″; what should be the value of O *o* so that the deflection of *n s* should be 15′? *Answer.* 209 centimetres, nearly.

(47.) A dipping needle makes 117 oscillations in a certain interval of time when oscillating *in* the plane of the magnetic meridian, and 98 oscillations in an equal interval of time when its plane of oscillation is perpendicular to the magnetic meridian. Find the angle of dip.

Let O A (Fig. 7) represent in magnitude and direction the total magnetic force of the earth at O, the plane of the paper being that of the magnetic meridian.

Draw OB horizontally and OC vertically, and complete the parallelogram OCAB.

Let angle BOA $= i$, so that $B\hat{O}A$ is the angle of *dip*.

Let $I = OA$ represent the total magnetic force of the earth at O,

$H = OB = OA \cos i =$ horizontal component of earth's force,

$V = OC = OA \sin i =$ vertical component.

If n represent the number of oscillations made by the needle in a given time when oscillating *in* the magnetic meridian, then we have

$$I = An^2,$$

where A is a constant whose value depends upon the nature of the particular magnet.

If now we compel the magnet to oscillate in a plane at right angles to the magnetic meridian, the horizontal component of the earth's force will have no influence on its motion, which will depend only on the vertical component V, and we shall have

$$V = I \sin i = An'^2;$$

$$\text{Hence } \sin i = \frac{V}{I} = \frac{n'^2}{n^2} = \left(\frac{98}{117}\right)^2 = \frac{9604}{13689}$$
$$= \sin 44^\circ\ 33'\ 15''.$$

(48.) The following observations were made by Humboldt. He counted the number of oscillations made by a dipping needle in a given time, first when its plane of oscillation coincided with, and then when it was perpendicular to, the magnetic meridian, with the following results:

Place.	In meridian	Perp. to meridian.
Quito	220	109
Mexico	242	209

Find the angle of dip at these two places.

Answer. 14° 12′ 35″ and 45° 51′ 21″.

(49.) The angle of inclination to the horizon of a dipping needle in a certain vertical plane (not that of the magnetic meridian) was 75° 39′, and in a vertical plane at right angles to the last it was 72°. What was the angle of *dip*?

Let O (see Fig. 8) be the centre of gravity of the dipping needle, O A the line of dip.

Let I = O A represent the total intensity of the earth's magnetic force.

Let O C be the vertical line in which the two given planes intersect, and O x, O y horizontal axes through O in each of these respective planes.

Let B O $x = \phi$ and A O $z = \theta$, then the resolved parts of the earth's magnetic force in the directions of the axes will be

$$X = I \sin\theta \cos\phi\,; \quad Y = I \sin\theta \sin\phi\,; \quad Z = I \cos\theta.$$

Then, if $\alpha = 75° \; 39'$, $\beta = 72°$,

$$\left.\begin{aligned} \cot\alpha &= \frac{X}{Z} = \frac{\sin\theta\cos\phi}{\cos\theta}, \\ \cot\beta &= \frac{Y}{Z} = \frac{\sin\theta\sin\phi}{\cos\theta}, \\ \cot i &= \tan\theta. \end{aligned}\right\}$$

whence

$$\begin{aligned} \cot^2 i = \tan^2\theta &= \cot^2\alpha + \cot^2\beta. \\ &= \cot^2 75° \; 39' + \cot^2 72° \\ &= \cdot 06545 + \cdot 10556 = \cdot 17101. \\ \therefore\; i &= 67° \; 32'. \end{aligned}$$

(50.) In another place a pair of similar observations in vertical planes at right angles with each other gave the angles 76° 30′ and 62° 18′; what was the angle of *dip*?

Answer. 60°.

(51.) A declination needle makes 50 oscillations in a minute at a place where the magnetic dip is 60°, and 57 oscillations in a minute at another place where the dip is

45°. Compare the intensities of the earth's magnetic field at these two places. *Answer.* As 1088 : 1000.

(52.) Suppose that when the magnetic dip in London is 67° 42′ a dipping needle be placed in such a position that its plane of motion makes an angle of 45° with the magnetic meridian. What angle will this needle make with the horizon?

Let θ = angle required; then, by the method of Example 49, we shall get

$$\tan\theta = \frac{\tan i}{\cos 45^\circ} = \frac{\tan 67^\circ\,42'}{\cos 45^\circ} = \tan 73^\circ\,50'.$$

(53.) Two small magnetised needles of exactly the same dimensions, but of slightly different magnetic power, when separately suspended were found to make 57 and 55 oscillations respectively under the influence of the earth's magnetic force. They were then mounted parallel to each other by a very light wire frame, and so that their similar poles were turned in opposite directions. The *astatic* combination was then suspended by a thread of unspun silk. What number of oscillations would it make per minute under the influence of the earth's magnetic force?

Let n_1 and n_2 be the number of oscillations in a second made by each needle separately, n = corresponding number when the two are mounted together.

Let f_1 = directive force exerted by the earth on the first magnet,
f_2 = directive force exerted by the earth on the second magnet,
K = moment of inertia of each magnet;

Then we have

$$f_1 = 4\pi^2 \mathrm{K} n_1^2, \text{ and } f_2 = 4\pi^2 \mathrm{K} n_2^2,$$
$$\therefore f_1 - f_2 = 4\pi^2 \mathrm{K}(n_1^2 - n_2^2).$$

Now, when the two small magnets are combined with their poles in opposite directions, the earth's directive force which acts on the combination is $f_1 - f_2$, and the moment of inertia of the system is 2 K, since the two needles have the same dimensions and the wire frame is very light; therefore

$$f_1 - f_2 = 4\pi^2 \, 2\,\text{K}\, n^2 = 4\pi^2 \text{K}(n_1^2 - n_2^2);$$

$$\therefore n^2 = \frac{n_1^2 - n_2^2}{2}; \ \therefore n = \sqrt{\frac{n_1^2 - n_2^2}{2}}$$

$$= \sqrt{\frac{57^2 - 55^2}{2}}$$

$$= 10{\cdot}58, \text{ nearly.}$$

(54.) If of two other needles of exactly similar dimensions one made 187 and the other 188 oscillations in a given time, what number of oscillations would be made in an equal interval of time by the pair if combined in the way described in the last example? *Answer.* 13·69.

RESISTANCE AND CONDUCTIVITY.

(1.) The legal *Ohm* is the resistance of a column of mercury one square millimetre in section and 106 centimetres in length, at the temperature of melting ice. What is the resistance between its ends of a layer of mercury 10 metres long, one centimetre wide, and ·125 centimetre thick?

The electrical resistance (R) of a conductor varies directly with its length (L), and inversely as its sectional area, and may be expressed in terms of these quantities by the equation

$$\text{R} = a\frac{\text{L}}{\text{S}},$$

where a is a constant quantity whose value depends upon the material of the conductor, and is numerically equal to the resistance of a band of this material whose length and cross section are each unity. It is called the *specific resistance* of the material.

Hence, if we have two wires whose lengths are l_1 and l_2, their sectional areas s_1 and s_2, their specific resistances a_1 and a_2, and their actual resistances R_1 and R_2, we shall have

$$R_1 = a_1 \frac{l_1}{s_1} \text{ and } R_2 = a_2 \frac{l_2}{s_2};$$

whence by division we get

$$\frac{R_2}{R_1} = \frac{a_2}{a_1} \times \frac{l_2}{l_1} \times \frac{s_1}{s_2},$$

and by means of this equation all problems connected with the relative resistances of wires in simple circuit can be easily solved.

In the present case $a_2 = a_1$, $l_2 = 1000$, $l_1 = 106$

$$s_2 = \cdot125, s_1 = \cdot01$$

$$R_1 = 1 \text{ ohm.}$$

$$\therefore R_2 = \frac{1000}{106} \times \frac{\cdot01}{\cdot125} = \cdot7547 \text{ ohm.}$$

(2.) What would be the resistance of a column of mercury one meter long and ·5 of a square millimetre in cross section? *Answer.* 1·8867 ohm.

(3.) Siemens' unit of resistance was that of a column of mercury one meter long and one square millimetre in section. What is this in ohms? *Answer.* ·9434 ohm.

(4.) What length of copper wire 4 millimetres in diameter would be equivalent to 12 metres of copper wire one millimetre in diameter?

In the fundamental equation we have

$$R_2 = R_1; \ l_2 = x; \ l_1 = 12; \ \frac{S_2}{S_1} = \frac{16}{1};$$

$$\therefore 1 = \frac{x}{12} \times \frac{1}{16}; \ \therefore x = 192 \text{ metres.}$$

(5.) A copper wire 6 metres long is found to have a diameter of ·74 millimetre; what will be the length of a copper wire of one millimetre diameter which will offer the same electrical resistance? *Answer.* 10·9569 metres.

(6.) Find the resistance of 15 miles of iron wire ·3 inch in diameter, having given that the resistance of one foot of iron wire ·001 inch in diameter is 59·1 ohms.

Answer. 52·008 ohms.

(7.) If the resistance of a German silver wire one metre long and one millimetre in diameter is ·2695 ohm, what length of German silver wire ·55 millimetre in diameter would be required to make up a resistance coil of 2 ohms?

Answer. 2·245 metres.

(8.) What would be the resistance of a telegraph line of 14 miles of iron wire ·238 inch in diameter?

Answer. 77·125 ohms.

(9.) Find the resistance at 0° C. of 20 metres of German silver wire weighing 52·5 grammes, having given that the resistance at 0° C. of a wire of this material one metre long and weighing one gramme is 1·85 ohm.

In this case

$$\frac{l_2\, s_2}{l_1\, s_1} = \frac{w_2}{w_1}, \quad \therefore \frac{s_1}{s_2} = \frac{l_2\, w_1}{l_1\, w_2},$$

and therefore the fundamental equation becomes

$$\frac{R_2}{R_1} = \frac{a_2}{a_1} \times \frac{l_2}{l_1} \times \frac{l_2}{l_1} \times \frac{w_1}{w_2} = \frac{a_2}{a_1} \times \left(\frac{l_2}{l_1}\right)^2 \times \frac{w_1}{w_2}.$$

$$\text{But, } a_2 = a_1; \ l_2 = 20; \ l_1 = 1; \ \frac{w_1}{w_2} = \frac{1}{52{\cdot}5}; \ R_1 = 1{\cdot}85,$$

$$\therefore R_2 = \frac{400 \times 1{\cdot}85}{52{\cdot}5} = 14{\cdot}095 \text{ ohms.}$$

(10.) A piece of silver wire three feet long was found to weigh 7·2 grains, and its resistance was ·3026 ohm. What would be the resistance of a wire one foot long, weighing one grain? *Answer.* ·2421 ohm.

(11.) What are the relative resistances of two copper wires, one of them 30·48 centimetres long and weighing 35 grammes, the other being 18·29 centimetres long and weighing 10·5 grammes? *Answer.* As 833 : 1000, nearly.

(12.) Find the resistance at 0° C. of 300 yards of iron wire weighing 54 pounds, having given that the resistance at 0° C. of an iron wire one foot long and weighing one grain is 1·0785 ohm. *Answer.* 2·311 ohms.

(13.) According to Matthiessen's experiments, the resistance at 0° C. of a piece of hard-drawn pure copper wire one foot long and weighing one grain is ·2106 ohm. If the resistance of a piece of ordinary copper wire 3 feet long and weighing 3·45 grains was found to be ·5782 ohm, compare the conducting power of this wire with that of a similar one of pure copper.

By the method employed in the solution of Example 9, we find that the resistance of a wire of pure copper of exactly the same dimensions as the second wire would be ·5494 ohm, and since the conducting power of a wire is the reciprocal of its resistance, we have

$$\frac{\text{conducting power of given wire}}{\text{conducting power of similar one of pure copper}} = \frac{\cdot 5494}{\cdot 5782}$$
$$= \cdot 95, \text{ nearly.}$$

∴ Conducting power of this specimen is about 95 per cent. of that of pure copper.

At the present day the conductivity of the copper wire supplied for telegraphic purposes is seldom less than 96 per cent. of that of pure copper.

The specific conductivity of the ordinary copper of commerce may vary from 44 to 101 per cent.

(14.) The resistance of 37 inches of a certain copper wire weighing 518 grains is found to be ·0041 ohm; calculate the conductivity of this wire as compared with pure copper. *Answer.* 94 per cent., nearly.

(15.) An iron wire weighing 249 pounds per mile and having a diameter of ·134 inch was found to have a resistance of 24·14 ohms per mile. Compare the conducting power of this wire with that of an exactly similar one of pure copper. *Answer.* 14 per cent., nearly.

(16.) Determine the specific resistance per cubic millimetre of a wire 437 millimetres long, which has a resistance of ·1257 ohm, and which weighs ·411 gramme in air and ·365 gramme in water.

The specific gravity of the wire $= \dfrac{\cdot 411}{\cdot 411 - \cdot 365} = 8\cdot 94.$

Its sectional area in square millimetres

$$= \frac{\cdot 411 \times 100}{43\cdot 7 \times 8\cdot 93} = \cdot 1053 \text{ sq. millimetre.}$$

The specific resistance per cubic millimetre being the resistance of a wire one millimetre long and one square millimetre in cross-section, will be

$$\cdot 1257 \times \frac{\cdot 1053}{4\cdot 37} = 3\cdot 0289 \times 10^{-5} \text{ ohm}$$
$$= 30\cdot 289 \text{ microhms.}$$

(17.) Find the specific resistance per cubic centimetre of the material of which the above wire was made.

Answer. 3·0289 microhms.

(18.) If the resistance of a platinum wire 3 metres long and weighing 3 grammes be 5·88 ohms, what will be its specific resistance per cubic centimetre, its specific gravity being 20·337. *Answer.* 9·6375 microhms.

(19.) If the specific resistance per cubic centimetre of a certain metal be 13·36 microhms, what will be the resistance of a wire of this material one metre long and 2 millimetres in diameter? *Answer.* 42526 microhms.

(20.) A wire 874 millimetres long and weighing ·822 gramme in vacuo and ·73 in water had a resistance of

·1257 ohm; find the specific resistance per cubic millimetre of the material of this wire.

Answer. 15·144 microhms.

(21.) A wire one foot long and ·001 inch in diameter has a resistance of 9·151 ohms; calculate the specific resistance per cubic inch of the material of the wire.

Answer. ·5989 microhm.

(22.) What must be the length of an iron wire whose sectional area is 4 square millimetres, if it is to have the same resistance as a copper wire 1000 yards long, whose cross-section is one square millimetre, assuming that the specific resistance of iron is 7 times that of copper?

In the equation of Example 1 we have

$$R_2 = R_1,\ l_2 = x,\ l_1 = 1000\,;$$

$$S_2 = 4,\ S_1 = 1,\ \frac{A_2}{A_1} = 7\,;$$

$$\therefore\ 1 = \frac{7}{1} \times \frac{x}{1000} \times \frac{1}{4}\,;$$

$$\therefore\ x = \frac{4000}{7} = 571\frac{3}{7} \text{ yards.}$$

(23.) How thick must an iron wire be which, for the same length, shall offer the same resistance as a copper wire 2·5 millimetres in diameter?

Answer. 6·61 millimetres.

(24.) For a line of telegraph iron wire is considered better than copper; what should be the thickness of the iron wire as compared with the copper so as to offer the same resistance? *Answer.* As 2·644 : 1.

(25.) A piece of copper wire is 10 inches long and of a certain thickness; what will be the length of a platinum wire of the same thickness which shall have the same resistance, taking the conductivity of platinum as $\frac{112}{1213}$ of that of copper? *Answer.* ·923 inch.

(26.) What should be the respective lengths of two wires of silver and lead so that they may each offer the same resistance as 10 inches of copper wire of the same thickness, the conductivity of silver and lead with respect to copper being 1·0467 and ·0923 respectively?

Answer. 10·467 inches and ·923 inch.

(27.) Find the resistance of 7 miles of copper wire ·238 inch in diameter, the resistance of one mile of copper wire ·2302 inch in diameter being equal to one ohm.

Answer. 6·5487 ohms.

(28.) What would be the resistance of 7 miles of iron wire ·238 inch in diameter, the relative resistance of this sample of iron with respect to copper being 7·5?

Answer. 49·115 ohms.

(29.) Find the resistance of 20 yards of platinum wire ·016 inch in diameter, the resistance of platinum with respect to that of copper being 11·3.

Answer. 26·581 ohms.

(30.) Find the resistance of a column of mercury one yard long and ·3 inch in diameter, the relative resistances of mercury and copper being as 50·7 : 1.

Answer. ·01696 ohm.

(31.) Assuming that the resistance of a copper wire one metre long and one millimetre in diameter is ·02104 ohm, and that 100 grammes of this copper are to be drawn into a wire which is to have a resistance of 100 ohms, what must be the length and diameter of this wire, the specific gravity of copper being 8·9?

Let x = length in metres of the required wire,
y = diameter in millimetres of the required wire.

Then we have

$$\frac{\pi y^2}{4} \times x \times 8{\cdot}9 = 100\,; \quad \therefore\ xy^2 = \frac{400}{\pi \times 8{\cdot}9} \quad . \quad . \quad . \quad (1).$$

Also by the formula in Example 1 we have

$$100 = {\cdot}02104 \times \frac{x}{y^2}; \;\therefore \frac{x}{y^2} = \frac{100}{{\cdot}02104} \quad . \quad . \quad . \quad (2)$$

Multiplying equations (1) and (2) together we get

$$x^2 = \frac{400 \times 100}{\pi \times 8{\cdot}9 \times {\cdot}02104}; \text{ whence } x = 260{\cdot}75 \text{ metres};$$

and

$$y^2 = \frac{{\cdot}02104 \times 260{\cdot}75}{100}; \text{ whence } y = {\cdot}234 \text{ millimetre.}$$

(32.) Fifty grammes of German silver wire are to be drawn into a wire which shall have a resistance of 1000 ohms. Find from the data given in Example 9 what must be the length of this wire. *Answer.* 164·39 metres.

(33.) How many metres of German silver wire 1·47 millimetre in diameter must be taken to give a resistance of one ohm? *Answer.* 8·018 metres.

(34.) Having given that the resistance of a wire made of German silver

at 0° C. is 1·850 ohm,
„ 10° C. „ 1·858229 „
„ 15° C. „ 1·862365 „

find the resistance of this wire at 13° C.

Let R_o = resistance of this wire at the temperature o° C.
R_t = „ „ „ t° C.

These values of the resistance at different temperatures may be connected by the equation

$$\frac{R_t}{R_o} = 1 + At + Bt^2;$$

where A and B are constants whose numerical values depend upon the particular wire, and are to be determined from the data of the problem.

Substituting we get

$$\frac{1{\cdot}858229}{1{\cdot}85} = 1 + 10\,A + 100\,B, \quad \text{or}$$

$$A + 10\,B = {\cdot}0004446\,;$$

$$\frac{1{\cdot}862365}{1{\cdot}85} = 1 + 15\,A + 225\,B, \quad \text{or}$$

$$A + 15\,B = {\cdot}0004456\,;$$

where $B = {\cdot}0000002$, and $A = {\cdot}0004426$.

$$\therefore R_{13} = 1{\cdot}85\{1 + 13 \times {\cdot}0004426 + 169 \times {\cdot}0000002\}$$

$$= 1{\cdot}85 \times 1{\cdot}0057876 = 1{\cdot}860707 \text{ ohms.}$$

(35.) Having given that the resistance of a piece of copper wire two metres long is

at	0° C. . . .	·2938 ohm,
„	7° C. . . .	·3016447 „
„	13° C. . . .	·3089835 „

find the resistance of this wire at 10° C.

Answer. ·30602 ohm.

(36.) The resistance of the copper wire of a cable when the temperature was 10° C. was found to be 2788 ohms; reduce this to the standard temperature of 24° C., the temperature coefficient being ·004097.

If R_o and R_t be the resistance of a certain wire at 0° C. and t° C. respectively, they may, very approximately, be connected by the relation

$$R_t = R_o\,(1 + K\,t),$$

where K is a constant which has a particular value for each metal. Hence we have

$$\frac{R_{t'}}{R_t} = \frac{1 + K\,t'}{1 + K\,t}\,;$$

and in the present case

$$\frac{R_{24}}{2788} = \frac{1 + 24 \times {\cdot}004097}{1 + 10 \times {\cdot}004097}\,; \text{ whence } R_{24} = 2941{\cdot}6 \text{ ohms.}$$

(37.) If the ordinary resistance of the copper conductor of a certain submarine cable is 2940 ohms at 24° C., and if on a certain occasion it is found to be 2849 ohms, what might we infer to be the average temperature of the sea bottom? *Answer.* 15°·7 C.

(38.) The resistance of a certain length of copper wire at 0° C. was 2728·5 ohms. What would be its resistance at 15° C.? *Answer.* 2896·2 ohms, nearly.

(39.) A solution of sulphate of copper was contained in a cylindrical glass vessel, the electrodes being circular copper plates 15 square centimetres in section placed horizontally, one at the bottom of the vessel and the other at some distance above it. It was then found that when the distance between the plates was 3·09 centimetres the resistance was 6 ohms. Find the resistance of a column of this sulphate of copper solution one metre long and one square millimetre in section.

By the method of Example 1 we get

$$R = 291262 \text{ ohms.}$$

(40.) What resistance would be offered by a solution of sulphate of copper to the passage of the current if the plates were each 10 square centimetres in area and at a distance of 10 centimetres? *Answer.* 29·1262 ohms.

(41.) In an experiment to determine the specific resistance of a saturated solution of common salt a small rectangular box was taken, the area of its cross section being 5·335 square inches, and was filled with the solution. This solution, a tangent galvanometer, and an adjustable resistance were included in circuit with a battery of constant strength, and the deflection of the galvanometer needle was 20°. The distance between the electrodes was then reduced by 8 inches, and in order to bring the deflection back to 20° an additional resistance of 1·644 ohm had to be introduced into the circuit. Assuming that one foot = 30·48 centimetres, find the specific resistance per cubic centimetre of this liquid. *Answer.* 2·7847 ohms.

N.B.—The object of the double observation in this experiment is to eliminate from the result the disturbing effect of the polarization of the electrodes.

(42.) A mixture was then made of 9 parts of a saturated solution of sulphate of copper and one part of a solution of common salt, and when the distance between the electrodes was reduced by 8 inches, the requisite additional resistance was 7·125 ohms. Find the specific resistance per cubic centimetre of the mixture.

Answer. 12·069 ohms nearly.

(43.) The box was then filled with pure spring water, when the resistance of the 8-inch prism was found to be 184·16 ohms. Find the specific resistance per cubic centimetre of the water. *Answer.* 311·942 ohms.

(44.) The box was then filled with distilled water, and the resistance of the 8-inch prism was found to be 5481 ohms. What was its specific resistance per cubic centimetre? *Answer.* 9284·1 ohms.

(45.) To the distilled water there was added ·5 per cent. of sulphuric acid, and this was found to reduce the resistance of the 8 inch prism to 5·48 ohms. Find the specific resistance per cubic centimetre. *Answer.* 9·28 ohms.

N.B. The results of Examples 44 and 45 illustrate the enormous increase in conductivity which is produced by acidulating the water in a voltameter.

BRANCH CIRCUITS AND SHUNTS.

(1.) An iron wire 20 metres long and ·2 centimetre in diameter, a copper wire 105 metres long and ·15 centimetre in diameter, and a German silver wire ·6 metre long and ·04 centimetre in diameter, were connected to two points, A and

B, in a circuit, (1) in series, (2) in multiple arc. Find the resistance of the circuit between A and B in each of these cases, having given

specific resistance of iron per cubic centimetre = 9·825 microhms;
specific resistance of copper per cubic centimetre = 1·616 microhms;
specific resistance of German silver per cubic centimetre = 21·170 microhms.

If two points, A and B, are connected by a series of n conducting wires, whose individual resistances are $r_1\ r_2 \ldots r_n$, then when these conductors are all in series the resultant resistance between A and B is

$$\text{R} = r_1 + r_2 + \ldots + r_n = \Sigma r.$$

But when each of the n wires has its ends connected immediately to A and B, then it is obvious that the conductivity of the circuit between A and B must be equal to the sum of the conductivities of the individual portions of this branch circuit, and since the conductivity of a wire is equal to the reciprocal of its resistance we must have

$$\frac{1}{\text{R}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots = \Sigma\frac{1}{r}.$$

By the formula in Example 1 of the last chapter we find that the values of r_1, r_2, r_3 for these several wires are, respectively,

for the iron wire $r_1 = 491250$ microhms,
„ copper wire $r_2 = 754133$ „
„ German silver wire $r_3 = 703875$ „

Hence, when all the wires are in series, we have for the resultant resistance between A and B,

$$\text{R} = r_1 + r_2 + r_3 = 2039258 \text{ microhms} = 2{\cdot}0393 \text{ ohms, nearly.}$$

When the wires are joined to A and B in multiple arc

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = 10^{-6}\{2\text{·}0356 + 1\text{·}3259 + 1\text{·}2596\}$$

$$= 10^{-6} \times 4\text{·}6211.$$

$$\therefore R = \frac{10^6}{4\text{·}6211} = 2\text{·}164 \times 10^5 \text{ microhms}$$

$$= 0\text{·}2164 \text{ ohm.}$$

(2.) Two points, A and B, in a circuit are connected in *multiple arc* by four wires, namely, one metre of iron wire one millimetre in diameter, 3 metres of German silver wire one millimetre in diameter, and 2 metres of platinum wire of the same diameter, and 10 centimetres of gold wire one millimetre in diameter. Find the resistance between A and B, having given that

specific resistance per cubic centimetre of gold wire = 2·081 microhms;

specific resistance per cubic centimetre of platinum wire = 9·158. *Answer.* 2558·3 microhms.

(3.) Four wires are joined together in multiple arc, their resistances being 38, 69, 57, and 21 ohms respectively. Find the resistance of the compound conductor thus formed.

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}} = \frac{1}{\text{·}0263 + \text{·}0145 + \text{·}0175 + \text{·}0476}$$

$$= \frac{1}{\text{·}1059} = 9\text{·}4429 \text{ ohms.}$$

(4.) Two points in a circuit are joined in multiple arc by two wires whose resistances are 21 and 61 ohms respectively. Find the resultant resistance of the circuit between these two points. *Answer.* 15·62 ohms, nearly.

(5.) Two points in a circuit are connected by 6 wires, whose individual resistances are 50, 100, 150, 200, 250, and 300 ohms. Find the resultant resistance between these two points. *Answer.* 20·4 ohms.

(6.) Three conducting wires whose individual resistances are 11, 3, and 8 ohms are joined in multiple arc. Find the resistance of this compound conductor.

Answer. 1·82 ohms.

(7.) An electric circuit consists of a generator whose resistance is 21 ohms, and an extrapolar portion consisting of two wires joined in multiple arc, whose respective resistances are 35 and 40 ohms. What is the resultant resistance of the whole circuit?

Answer. $R = 21 + 18\frac{2}{3} = 39\frac{2}{3}$ ohms.

(8.) What would have been the resultant resistance of the whole circuit if the extrapolar portion had consisted of 4 separate wires, whose resistances were 35, 40, 45, and 50 ohms respectively? *Answer.* 31·44 ohms, nearly.

(9.) The total resistance of an electric circuit is known to be 50 ohms, and on connecting two points of the circuit, A and B, by an additional conductor, the resistance of the circuit was found to be 43·3 ohms. The original resistance between A and B being 10 ohms, what was the resistance of the added wire?

Let x = resistance of additional wire in ohms.

Then $\frac{1}{10} + \frac{1}{x} = \frac{1}{3{\cdot}3}$; whence $x = \frac{330}{67} = 5$ ohms, nearly.

(10.) The resistance between two points A and B of a circuit was known to be 33 ohms, but on adding another wire between A and B, the resistance fell to 25 ohms. What was the resistance of this wire?

Answer. 103·125 ohms.

(11.) If a galvanometer, whose resistance is 2641 ohms, is shunted by a shunt of 100 ohms resistance, what will be the resistance of the shunted galvanometer?

A *shunt* is a piece of wire placed across the poles of a galvanometer so as to reduce its sensibility by allowing a portion of the current to pass through the shunt wire.

$$\left.\begin{array}{l} \text{Let } G = \text{resistance of galvanometer,} \\ \phantom{\text{Let }} S = \phantom{\text{resistance}}\text{,, shunt,} \\ \phantom{\text{Let }} R = \phantom{\text{resistance}}\text{,, shunted galvanometer.} \end{array}\right\}$$

$$\text{Then } \frac{1}{R} = \frac{1}{G} + \frac{1}{S};$$

$$\therefore R = \frac{G \times S}{G + S} = \frac{2641 \times 100}{2641 + 100}$$

$$= 96{\cdot}35 \text{ ohms.}$$

(12.) A galvanometer whose resistance was 74345 ohms was shunted by a shunt of 100 ohms resistance. Find the resistance of the shunted galvanometer.

Answer. 99·87 ohms nearly.

(13.) A shunt of ·9079 ohm was applied to a galvanometer of 907 ohms resistance. What was the resistance of the shunted galvanometer? *Answer.* ·907 ohm.

(14.) Compare the sensibilities of the shunted and unshunted galvanometers in the last example.

$$\frac{\text{Sensibility of unshunted galvanometer}}{\text{Sensibility of shunted galvanometer}} = \frac{G + S}{S}$$

$$\frac{907{\cdot}9079}{{\cdot}9079} = \frac{1000}{1} \text{ nearly.}$$

(15.) Compare the sensibility of the shunted and unshunted galvanometer in Example 11.

Answer. As 27·41 : 1.

(16.) A certain mirror galvanometer has a resistance of 8432 ohms, and we require for it a *shunt* which will reduce its sensibility a hundredfold. What must be the resistance of the shunt? *Answer.* 85·2 ohms.

(17.) A galvanometer whose resistance is 246 ohms has to be supplied with a $\frac{1}{50}$th and a $\frac{1}{100}$th shunt. What must be the resistances of each of these shunts?

Answer. 5·02 and 2·485 ohms respectively.

Note.—Students are occasionally confused by the fact that a shunt of given resistance is called by some persons a $\frac{1}{10}$th shunt and by others a $\frac{1}{9}$th shunt.

When a given shunt is called a $\frac{1}{9}$th shunt it means that $\frac{\text{resistance of shunt}}{\text{resistance of galvanometer}} = \frac{1}{9}$, but when it is called a $\frac{1}{10}$th shunt, reference is made to the fact that $\frac{\text{current in galvanometer}}{\text{current in main line}} = \frac{1}{10}$.

(18.) It is required to reduce the sensibility of a galvanometer of 3·75 ohms resistance a hundredfold. What must be the resistance of the shunt?

Answer. ·0379 ohm nearly.

(19.) A galvanometer whose resistance was 5000 ohms was shunted by a shunt of 50·5 ohms resistance; but, in order to compensate for the diminished resistance of the galvanometer, an additional resistance was introduced into the circuit. How great was it? *Answer.* 4950 ohms.

(20.) If the shunt to a galvanometer of 236 ohms resistance reduced its sensibility to $\frac{1}{5}$th, what additional resistance would have to be introduced into the circuit so as to compensate for the reduced resistance of the galvanometer?

Answer. 189 ohms.

CURRENTS IN SIMPLE CIRCUITS.

(1.) The current from a battery of 50 cells in series, each of them having an electromotive force of 1·12 volt and an internal resistance of 9 ohms, is sent through a galvanometer whose resistance is 246 ohms and an extrapolar resistance of 100 ohms. What is the current strength?

Let C represent the strength of the current in ampères,
ΣE represent the algebraical sum of all the effective electromotive forces in the circuit expressed in volts,
ΣR represent the sum of all the resistances in the circuit in ohms.

Then by Ohm's law we have

$$\text{C} = \frac{\Sigma \text{E}}{\Sigma \text{R}} = \frac{50 \times 1{\cdot}12}{50 \times 9 + 246 + 100} = \frac{56}{796}$$
$$= {\cdot}07 \text{ ampère nearly.}$$

(2.) A battery of 30 cells was connected up in series with an extrapolar resistance of 200 ohms and a galvanometer of very small resistance. The electromotive force of each cell being 1·9 volt, and its internal resistance 6 ohms, what was the current strength? *Answer.* ·15 ampère.

(3.) A battery of 15 cells was connected up in series with a line of 300 ohms resistance. Find the current strength if the average resistance of each cell was 9·42 ohms and its electromotive force 1·61 volts.

Answer. ·0547 ampère nearly.

(4.) The same battery was joined up to a galvanometer of 110 ohms resistance and an extrapolar resistance of 150 ohms. What was the current strength?

Answer. 60·179 milliampères.

(5.) The battery in Example 2 was then arranged in two series of 15 cells, each with the same extrapolar resistance. What was the current and strength in the extrapolar portion of the circuit?

With respect to the resistance of the battery, each pair of cells is now equivalent to a single cell of the same electromotive force as before but of twice the effective area, and consequently of one-half the original resistance. Hence we shall have

resistance of battery = 15 × 3 = 45 ohms,
effective electromotive force = 15 × 1·9 = 28·5 volts,

$$\therefore \text{C} = \frac{\Sigma \text{E}}{\Sigma \text{R}} = \frac{28{\cdot}5}{45 + 200} = {\cdot}11633 \text{ ampère, nearly.}$$

(6.) Again, the same battery was arranged in three series of 10 cells each with the same extrapolar resistance. What was the strength of current in the extrapolar portion of the circuit? *Answer.* 86·364 milliampères.

(7.) What would be the ratio of the strengths of the currents produced by a Leclanché cell of electromotive force 1·26 volt and 4 ohms resistance, if its poles were connected in one case by a wire of 0·5 ohm, and in another case by a wire of 100 ohms resistance?

Answer. As 23 : 1 nearly.

(8.) Compare the currents which the same electromotive force is capable of producing in two wires of the same material whose lengths are as 5 : 1 and their cross sections as 3 : 2. *Answer.* As 3 : 10.

(9.) Find the strength of current sent by a battery of 10 cells, each of which has an internal resistance of 4 ohms and an electromotive force of 1 volt, through an extrapolar resistance of 2000 ohms. *Answer.* $\frac{1}{204}$ ampère.

(10.) What would have been the strength of the current if the area of each of the battery-plates had been doubled?

Answer. $\frac{1}{202}$ ampère.

(11.) Supposing, however, that the number of cells were doubled, their size remaining as in Example 9, what would be the strength of the current through the same extrapolar resistance? *Answer.* $\frac{1}{104}$ ampère.

(12.) Assuming that the electromotive force of a Daniell's cell is to that of a Grove's cell as 1 : 1·75, and that the resistances are as 2 : 1, compare the strengths of the currents which will be produced (1) in a circuit of 5 Grove's cells in series, and an extrapolar resistance equivalent to that of 10 such cells; (2) in a circuit consisting of 10 Daniell cells and an extrapolar resistance equivalent to that of 5 such cells. *Answer.* As 7 : 4.

(13.) Compare the strengths of the currents which could be sent through the same telegraphic circuit of 80 ohms resistance by two different batteries, one composed of 30 cells, each of which has a resistance of 5 ohms and an electromotive force of 1·37 volts, and the other battery consisting of 25 cells, each of which has a resistance of 1·5 ohm and an electromotive force of 1·86 volt.

Answer. As 9 : 20 nearly.

(14.) A battery of 10 cells, each of which had an electromotive force of 1·12 volts and an internal resistance of 4 ohms, was arranged in series and joined to another battery of 20 cells, each of which had an electromotive force of ·9 volt and a resistance of 7·5 ohms, so that the like poles of the two batteries were connected. What was the strength of current in the circuit?

The electromotive forces of the batteries are here opposed to each other.

Hence

$$\Sigma \text{E} = 20 \times \cdot 9 - 10 \times 1 \cdot 12 = 6 \cdot 8 \text{ volts,}$$
$$\Sigma \text{R} = 4 \times 10 + 20 \times 7 \cdot 5 = 190 \text{ ohms,}$$
$$\therefore \text{C} = \frac{\Sigma \text{E}}{\Sigma \text{R}} = \frac{6 \cdot 8}{190} = 3 \cdot 5789 \times 10^{-2} \text{ ampère.}$$
$$= 35 \cdot 8 \text{ milliampères nearly.}$$

(15.) A battery consisting of 15 equal cells, each of which had an electromotive force of 1·008 volts and an internal resistance of 1·25 ohm, was joined up with another battery consisting of a series of 12 cells, each of which had an electromotive force of 1·12 volts and an internal resistance of 7 ohms, so that the like poles were connected. Find the strength of current flowing in this circuit.

Answer. 16·35 milliampères.

(16.) The receiving instrument at a telegraph station has a resistance of 125 ohms and requires a current of 10 milliampères to work it. The line resistance is 260 ohms. Of how many cells, in series, must the sending battery consist if each of them has a resistance of 2 ohms and an electromotive force of 1·08 volts?

Let x = required number of cells, then the battery resistance is $2x$ ohms and the electromotive force is $x \times 1 \cdot 08$ volts. Hence, by Ohm's law,

$$C = \frac{x \times 1{\cdot}08}{125 + 260 + 2x} = {\cdot}01 \text{ ampère, by the question;}$$

$$\therefore \frac{x \times 1{\cdot}08}{2x + 385} = {\cdot}01, \ \therefore x = 3{\cdot}632;$$

and the nearest integer being 4, this will be the number of cells required, and they will give a current of about 11 milliampères.

(17.) The resistance of a telegraph line between Paris and Strasbourg was 5000 ohms, and that of the electro-magnet of the receiving instrument 2000 ohms. How many Daniell's cells would be required if the resistance of each cell is 10 ohms, its electromotive force 1 volt, and that the minimum current which will work the receiving instrument is 5·4 milliampères? *Answer.* 40 cells.

(18.) What will be the strength of current which can be sent through a line of copper wire 400 miles long and ·238 inch in diameter by a battery of 50 Daniell's cells in series, each of which has an electromotive force of 1·12 volt and a resistance of 5 ohms, assuming that the resistance of a copper wire one foot long and ·001 inch in diameter is ·9·94 ohms? *Answer.* 90·23 milliampères.

(19.) Compare the strength of the current produced by two equal cells in series in a wire whose resistance is equal to $\frac{1}{20}$th of the resistance of either cell, with that produced by 5 similar cells in the same wire. Also compare the currents produced by the same batteries in a wire whose resistance is equal to 200 times that of a single cell.

Answer. (1) As 202 : 205; (2) As 41 : 101.

(20.) An individual was placed on an insulating stool and took hold of the poles of a battery whose electromotive force was 100 volts and its resistance 10·1 ohms. Supposing that the electrical resistance of this person was 989·9 ohms, what was the strength of the current flowing through his body? *Answer.* 100 milliampères.

(21.) The resistance of a Thomson's mirror galvanometer is sometimes as great as 50,000 ohms, and such an instrument will give a deflection of 200 divisions of the scale with a single Daniell's cell through a line resistance of 10 megohms. Assuming the electromotive force of a Daniell cell to be 1 volt, what would be the strength of current flowing through the galvanometer?

Answer. $\frac{1}{10050}$ of a milliampère.

(22.) The same current was sent simultaneously through two tangent galvanometers A and B, producing in them the deflections 28° 20′ and 21° respectively. If on another occasion a current of a certain strength produce a deflection of 37° in the galvanometer A, what deflection would the same current have produced in B?

When a tangent galvanometer is inserted in a circuit with the plane of its coil coinciding with that of the magnetic meridian, a current of strength, c, will produce a deflection, θ, the two being connected by the equation

$$c = m \tan \theta,$$

where m is a constant whose numerical value depends upon the construction of the particular instrument and upon the intensity of the local magnetic field.

Let m_1 and m_2 be the values of the constant m for the two galvanometers A and B; then from the first experiment we have

$$m_1 \tan 28^\circ\ 20' = m_2 \tan 21^\circ$$

$$\therefore \frac{m_1}{m_2} = \frac{\tan 21^\circ}{\tan 28^\circ\ 20'} \quad . \quad . \quad . \quad (1)$$

Let x° = the number of degrees of deflection of B when the second current is passing through it, then we shall have

$$\frac{m_1}{m_2} = \frac{\tan x^\circ}{\tan 37^\circ} \quad . \quad . \quad . \quad . \quad (2)$$

From equations (1) and (2)

$$\tan x = \frac{\tan 37^\circ \times \tan 21^\circ}{\tan 28^\circ\ 20'};$$

$$\therefore x = 28^\circ\ 12'\ 44''.$$

(23.) The current from a battery (A) when sent through a certain tangent galvanometer produces a deflection of 18° 30′, while the current from another battery (B) on being sent through the same galvanometer produces a deflection of 26°. Compare the strengths of the two currents.

Answer. As 69 : 100, nearly.

(24.) A tangent galvanometer of very small resistance was included in a circuit consisting of a series of 12 cells and a wire resistance of 1000 ohms and the deflection of the needle was 13°. Had the cells been arranged in two parallel series of 6 each, what would have been the deflection of the needle? The resistance of each cell was 2 ohms, and its electromotive force 1 volt.

By Example 22 $C = m \tan\theta = \frac{\Sigma E}{\Sigma R}$ by Ohm's law.

But $\Sigma E = 12$ volts, and $\Sigma R = 12 \times 2 + 1000 = 1{,}024$ ohms; $\theta = 13°$.

Hence, for the value of m, to which the name *reduction factor* of the galvanometer was given by Mueller, we have

$$m = \frac{12}{1024 \times \tan 13°} = \frac{1}{19{\cdot}7} \text{ nearly.}$$

When the cells are arranged in two series of 6 each we have $\Sigma E = 6$ volts, and $\Sigma R = 6 + 1000 = 1006$ ohms.

$$\text{Then } \tan\theta = \frac{1}{m} \times \frac{\Sigma E}{\Sigma R} = \frac{19{\cdot}7 \times 6}{1006} = {\cdot}1175\,;$$

$$\text{whence } \theta = 6° \; 42'.$$

(25.) With the same galvanometer, a wire resistance of 500 ohms, and the battery arranged in three parallel series of 4 each; what would have been the deflection of the needle?

Answer. 8° 55′.

(26.) If the battery had been arranged in six parallel series of 2 cells each, what would have been the deflection of the galvanometer needle? *Answer.* 4° 30′.

(27.) If with another galvanometer, the battery and resistance which were described in Example 24, gave a deflection of 12°, what was the value of the *reduction factor*?

Answer. $\frac{1}{18\cdot138}$.

(28.) A *sine* galvanometer had a resistance of ½ an ohm, and was included in circuit with a cell whose electromotive force was 1 volt and its resistance 1·4 ohm, and a wire resistance of 20 ohms, when the coil had to be turned through 30° to overtake the needle. A length of 20 metres of iron wire, one millimetre in diameter, was then added to the circuit, when the requisite displacement of the coil from the magnetic meridian was 26° 48′. What was the resistance of this piece of iron wire?

In using a sine galvanometer the plane of the coil is initially *in* the magnetic meridian, and when the current in the coil deflects the needle, the plane of the coil is twisted round until the needle is again in its initial position with respect to the coil.

Let m represent the *reduction factor* of this galvanometer; then

$$\text{C} = m \sin \theta,$$

and, by the method employed in Example 22, we shall find from the first part of the problem that

$$m = \frac{1}{10\cdot95} = \frac{1}{11}, \text{ nearly.}$$

Let x = resistance, in ohms, of the iron wire, then by Ohm's law

$$\frac{1}{x + 20 + 1\cdot4 + \cdot5} = \frac{\sin 26^\circ\ 48'}{11} = \frac{\cdot4509}{11};$$

$$\therefore\ x + 21\cdot9 = \frac{11}{\cdot4509} = 24\cdot4, \text{ nearly};$$

$$\therefore\ x = 2\cdot5 \text{ ohms.}$$

(29.) A piece of platinum-silver wire was then substituted for the iron wire, and the requisite displacement of the coil was 22° 54′. What was the resistance of the wire?

Answer. 6·4 ohms, nearly.

(30.) What would have been the requisite displacement of the coil in the case of the first part of Example 28 if a resistance of 40·6 ohms had been substituted for the 20 ohms?

Answer. 14° 56′.

(31.) In using a certain reflecting galvanometer the mirror was at a distance of one metre from the scale, which was 40 centimetres long, and was divided into millimetres. What was the angular displacement of the mirror when the deflection of the spot of light was 15 millimetres?

It is proved in treatises on Optics, that if θ is the angular displacement of the mirror, the corresponding displacement of the reflected beam is $2\,\theta$; and in the present case

$$\tan 2\,\theta = \tfrac{15}{1000} = \cdot015\ ; \ \therefore\ \theta = 25'\ 46''.$$

(32.) If the reflected spot of light had moved just to the end of the scale, that is to say through 20 centimetres, what would have been the corresponding deflection of the magnet attached to the mirror? *Answer.* 5° 39′.

(33.) Compare the strengths of two currents which, on being sent successively through the above-mentioned reflecting galvanometer, cause the spot of light to be deflected 10 and 15 millimetres from its central position.

Let θ_1 and θ_2 be the corresponding angular displacements of the mirror;

$$\text{then}\ \left.\begin{aligned}\tan 2\,\theta_1 &= \ \cdot01\\ \tan 2\,\theta_2 &= \cdot015\end{aligned}\right\}; \text{ whence } \left.\begin{aligned}\theta_1 &= 17'\ 30''\\ \theta_2 &= 26'\ 15''\end{aligned}\right\};$$

and if c_1 and c_2 represent the strengths of the currents,

$$\frac{c_2}{c_1} = \frac{\tan 26'\ 15''}{\tan 17'\ 30''} = \tfrac{3}{2},\ \text{very nearly.}$$

Hence, for small displacements the strengths of the currents are directly proportional to the displacements of the spot of light upon the screen.

(34.) On another occasion a current was sent through this galvanometer and caused the spot of light to travel just to the end of the scale, while another current caused a deflection of 10 centimetres. Compare the strengths of these two currents.

Answer. As 1983 : 1000; or very nearly as the deflections.

(35.) A microphone has a resistance of 20 ohms, and the sound of the human voice diminishes this by a mean amount of 2 ohms. It is connected with a telephone by a double line of copper wire, the resistance of which is 25 ohms per mile, and a battery of 2 cells, each of which has an electromotive force of 1 volt and a resistance of 2 ohms. The inducing wire of the telephone has a resistance of 1 ohm and requires a minimum variation of current of ·01 milliampère to make it speak. What will be the maximum distance at which this telephone can work?

Let $x =$ number of miles; then $2x \times 25 =$ line resistance.

Also $\Sigma \text{E} = 2$ volts; and when the microphone is silent,

$$\Sigma \text{R} = 50x + 4 + 1 + 20 = 50x + 25 \text{ ohms};$$

$$\therefore \text{ current through telephone } = \frac{2}{25(2x+1)} = \text{C}_1.$$

Again, when the microphone is spoken to its resistance is 18 ohms, and in that case,

$$\Sigma \text{R} = 50x + 23;$$

$$\therefore \text{C}_2 = \frac{2}{50x + 23}.$$

And in order for the telephone to speak, the conditions of the problem give us

$$\frac{2}{50x+23} - \frac{2}{50x+25} = \frac{1}{100000};$$

whence $x = 12{\cdot}3$ miles nearly.

(36.) In a circuit whose total resistance, exclusive of the microphone, was 72 ohms, were a Leclanché cell and a tangent galvanometer. When the resistance of the microphonic contact was 11·2 ohms the ticking of a watch could just be heard in the telephone. Assuming that the requisite minimum variation of current was the same as in the last example, what was the change in the resistance of the microphonic contact produced by the ticking of the watch?

Let x = required change in resistance; then we have $E = 1·26$ volt.

$$\left.\begin{array}{l} R_1 = 83·2 \quad \text{ohms} \\ R_2 = 83·2 - x \quad ,, \end{array}\right\}; \quad \therefore 1·26\left\{\frac{1}{83·2 - x} - \frac{1}{83·2}\right\} = \frac{1}{100000};$$

$$\therefore x = ·0549 \text{ ohm.}$$

CURRENTS IN BRANCH CIRCUITS.

(1.) The electromotive force of a battery was 10 volts, and its internal resistance was 20 ohms. The extrapolar circuit consisted of 4 separate wires connected to the battery in multiple arc, their resistances being 35, 40, 45 and 50 ohms respectively. Find the strength of the current circulating in the battery and in each of the extrapolar wires.

Let r_1, r_2, r_3, r_4 represent the resistances of each of the wires, and B that of the battery; then, if R represent the resultant resistance of the extrapolar portion of the circuit,

$$\begin{aligned} \frac{1}{R} &= \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} \\ &= \tfrac{1}{35} + \tfrac{1}{40} + \tfrac{1}{45} + \tfrac{1}{50} \\ &= ·0286 + ·0250 + ·0222 + ·0200 \\ &= ·0958; \end{aligned}$$

$$\therefore R = 10·4384 \text{ ohms};$$

$$\therefore \Sigma R = B + R = 20 + 10·4384 = 30·4384.$$

Let c, c_1, c_2, c_3, c_4, represent the currents in the battery and in each of the four extrapolar wires respectively; then by Ohm's law

$$c = \frac{\Sigma E}{\Sigma R} = \frac{10}{30{\cdot}4384} = {\cdot}3285 \text{ ampère.}$$

In the extrapolar portion of the circuit the current always divides itself between the conductors in direct proportion to their several conductivities;

$$\therefore \frac{c_1}{c} = \frac{\frac{1}{r_1}}{\frac{1}{R}} = \frac{R}{r_1}; \therefore c_1 = c \times \frac{R}{r_1} = \frac{{\cdot}3285 \times 10{\cdot}4384}{35}$$

$$= {\cdot}098 \text{ ampère}$$

$$= 98 \text{ milliampères, nearly.}$$

Similarly we shall find that

$$c_2 = 85{\cdot}7 \text{ milliampères,}$$
$$c_3 = 76{\cdot}2 \quad \text{,,}$$
$$c_4 = 68{\cdot}6 \quad \text{,,}$$

N.B.—The student will notice that $c_1+c_2+c_3+c_4=c$; and this affords a means of checking the values for the several partial currents.

(2.) The extrapolar portion of an electric circuit consists of two wires joined to the battery in multiple arc, their resistances being 30 and 25 ohms respectively. Supposing that a current of one ampère is circulating in the battery, what will be the strength of the current in each of these wires? *Answer.* $\frac{5}{11}$ and $\frac{6}{11}$ ampère.

(3.) The poles of a battery through which a current of one ampère is flowing are joined by three wires in multiple arc whose respective resistances are 30, 50, and 70 ohms. What is the current strength in each of these wires?

Answer. $\frac{35}{71}$, $\frac{21}{71}$, and $\frac{15}{71}$ of an ampère.

(4.) A couple of Daniell's cells, each of which had an electromotive force of 1·079 volt and a resistance of 2 ohms, were joined in series, and their opposite poles were connected by a wire of 10 ohms resistance and by an electromagnet which, with its connecting wires, had a resistance of 10 ohms. What was the strength of the current in the electromagnet? *Answer.* 119·9 milliampères.

(5.) If the single wire had a resistance of 100 ohms, what would have been the current in the electromagnet?

Answer. 150 milliampères.

(6.) If the resistance of the single wire had been 1 ohm, what would have been the current in the electromagnet?

Answer. 39·6 milliampères.

(7.) Three electromagnets, whose resistances with their connecting wires were 60, 80·5, and 12·2 ohms respectively, were joined in multiple arc to the poles of a battery through which a current of one ampère then flowed. What was the strength of the current in each?

Answer. 150·5, 111·7, and 737·8 milliampères, approximately.

(8.) In a similar case the resistance of the three extrapolar portions of the circuit were 520, 335, and 210 ohms respectively, and a current of 10 ampères flowed through the battery. What was the strength of the current in each of the branches?

Answer. 1·989, 3·087, 4·924 ampères, approximately.

(9.) The extrapolar portion of a certain circuit consisted of 150 metres of copper wire, 1 millimetre in diameter, which had a resistance of 3·366 ohms. The resistance of the battery was ·85 ohm, and it was found that a current of one ampère was circulating. The poles of the battery were then connected in multiple arc by a second copper wire 250 kilometres long and 5 millimetres in diameter. What was then the strength of the current flowing in each of the extrapolar wires?

To determine the electromotive force of the battery, we have from the first part of the question

$$c_1 = \frac{E}{3\cdot366 + \cdot85} = \frac{E}{4\cdot216} = 1 \text{ by the question;}$$
$$\therefore E = 4\cdot216 \text{ volts;}$$

and then by the method of Example 1 we get from the second part of the question

$$\left.\begin{array}{l}\text{current in short wire} = 997 \text{ milliampères} \\ \quad\text{,,} \qquad \text{long} \quad \text{,,} \;\; = \;\; 15 \qquad\quad \text{,,}\end{array}\right\}, \text{ approximately.}$$

(10.) Suppose that in the last example we replaced the original 150-metre wire by an iron wire 25 kilometres long and 5 millimetres in diameter. What would be the strengths of the currents in each of the extrapolar wires if we assume the resistance of iron to be 7 times that of copper?

Answer. 18·6 and 26·5 milliampères.

(11.) The extrapolar portion of a voltaic circuit consists of 10 wires joined in multiple arc, the resistance of each wire being 10 ohms. Taking the resistance of the battery equal to 10 ohms and its electromotive force at 10 volts, find the strength of the current in each of the extrapolar wires. *Answer.* $\frac{1}{11}$th of an ampère.

(12.) In order to maintain a current of a definite strength in a certain circuit, a wire was introduced into the circuit having a resistance of 2·914 ohms, and the ends of this wire were also connected with a branch circuit consisting of a galvanometer and one of Latimer Clark's standard cells whose electromotive force is 1·457 volt. The strength of the current in the main circuit was then adjusted so that the galvanometer showed no deflection. What was the current strength?

Let c_1 be the strength of the current in the trunk circuit (see fig. 9), and c_2 and c_3 the currents in the branches. Then by Kirchhoff's laws we have at the point A

$$c_1 + c_2 - c_3 = 0.$$

But $c_2 = o$ by hypothesis; $\therefore c_1 = c_3$; that is to say, whatever may be the strength of current through R, the same will be the strength of current in the trunk circuit.

But strength of current in branch R

$$= \frac{\text{difference of potential at A and B}}{\text{resistance of R}} = \frac{E}{R} \text{ ampères.}$$

Also, since the electromotive force of the Clark cell in the branch A G B just prevents any of the trunk current coming down that branch, it is evident that the difference of potentials at A and B must be equal to the difference of potential of the electrodes of a Clark's cell—*i.e.* 1·457 volt;

$$\therefore c_1 = c_3 = \frac{1{\cdot}457}{2{\cdot}914} = {\cdot}5 \text{ ampère.}$$

(13.) An experimenter desired to maintain a constant current of 5 ampères through a certain circuit. For this purpose he introduced into the circuit a wire having a resistance R, and the ends of this wire were also connected with a branch circuit containing one of Clark's cells and a galvanometer, and the resistance R was adjusted till there was no current flowing through the galvanometer. What was the value of R? *Answer.* ·2914 ohm.

(14.) A, B, C, D is a Wheatstone bridge arrangement of conductors (see fig. 10), the galvanometer being in B D and the resistance whose value is required in B C. The resistances in A B and A D are 10 and 100 ohms, and the resistance of D C is 327 ohms. What is the value of the required resistance?

The theory of the *bridge* is generally proved from Kirchhoff's laws, and the student will find it in any good text-book on electricity. He may also find it useful to consider it in the following way.

Suppose B and D not connected, and that the potentials at A and C are V and V′, while the potentials at B and D are

v_1 and v_2. Let the resistances of A B, A D, B C, D C be represented by R_1, R_2, R_3, R_4; then by Ohm's law we must have

$$\frac{V - v_1}{R_1} = \frac{v_1 - V'}{R_3}, \text{ and } \frac{V - v_2}{R_2} = \frac{v_2 - V'}{R_4}.$$

From these equations we get

$$v_1 = \frac{V R_3 + V' R_1}{R_1 + R_3}, \text{ and } v_2 = \frac{V R_4 + V' R_2}{R_2 + R_4};$$

and if no current is to flow through B D when the points B and D are connected, we must have $v_1 = v_2$, and in that case

$$\frac{V R_3 + V' R_1}{R_1 + R_3} = \frac{V R_4 + V' R_2}{R_4 + R_2};$$

whence

$$(V - V')(R_2 R_3 - R_1 R_4) = 0.$$

But since the current in the trunk circuit is not zero, $V - V'$ cannot be zero, and ∴ we must have

$$R_2 R_3 = R_1 R_4.$$

In the present case

$$R_1 = 10,\ R_2 = 100,\ R_4 = 327 \text{ ohms};$$

$$\therefore R_3 = \frac{R_1 R_4}{R_2} = \frac{327 \times 10}{100} = 32{\cdot}7 \text{ ohms.}$$

(15.) What should be the resistance of the galvanometer employed in the experiment described in the last example so as to obtain the maximum sensibility, if it be known that the required resistance is about 32·7 ohms?

It can easily be proved that the magnetic moment of the galvanometer will be raised to its maximum when the resistance of the galvanometer is equal to the resultant resistance of B A D and B C D taken parallel between B and D.

In the present case we shall have

$$G = \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} = \frac{110 \times 359{\cdot}7}{469{\cdot}7} = 84{\cdot}239$$
$$= 84 \text{ ohms, nearly.}$$

(16.) Some resistances, all of which are known to be very near 1000 ohms, are to be measured with a bridge in which the arms A B, A D, are each 100 ohms. Out of a stock of galvanometers of different resistances, which would be the best to use?

Answer. The one whose resistance is nearest to 182 ohms.

(17.) Suppose that the galvanometer has a resistance of 246 ohms, that the unknown resistance is somewhere about 424·2 ohms, and that we want to know its value as near as we can to the $\frac{1}{100}$th of an ohm. What would be the best values for R_1 and R_2?

In the formula of Example 14 we have

$$G = 246 \text{ ohms}, \ R_2 = 100\ R_1\ ; \ R_4 = 100, \ R_3 = 42420\ ;$$

and then by Example 15 we get

$$R_1 = 247{\cdot}4 \text{ and } R_2 = 2{\cdot}474 \text{ as the best values.}$$

(18.) If the galvanometer had a resistance of 20 ohms, and the required resistance was known to be very nearly 60 ohms, what would be the best values to give to R_1 and R_2 supposing them equal? *Answer.* 12 ohms.

(19.) In a common form of Wheatstone bridge the resistances R_2 R_4 are formed of a single piece of wire along which the contact piece D is made to travel until the balance is obtained. Suppose that R_1 and R_3 were each 5 ohms, and that A D C consisted of a piece of German silver wire whose resistance was ·27 ohm, what should be the resistance of the galvanometer so as to give the best results?

Answer. 2·57 ohms, nearly.

(20.) In the form of bridge mentioned in the last example R_1 was 5 ohms, and the lengths of A D, D C, when the balance was obtained, were 53 : 47. What was the value of the resistance R_3? *Answer.* 4·434 ohms.

SPECIAL GROUPING OF CELLS.

(1.) Suppose that we have 24 Daniell's cells, each of which has an electromotive force of 1 volt and an internal resistance of 2 ohms. What will be the best way of grouping these cells so as to send the maximum current through an electromagnet whose resistance, including that of the leading wires, is 4 ohms?

Let B represent the resistance of all the cells together when in single file.

Let R represent the resistance of the extra-polar portion of the circuit.

Let x represent the number of files of cells.

Let E represent the electromotive force of all the cells when in single file.

Then the resultant electromotive force of battery $= \frac{E}{x}$;

" " resistance of battery $= \frac{B}{x} \times \frac{1}{x}$;

because each row is equivalent to a cell whose plates are x times as large as they would be if all the cells were in single file. Then by Ohm's law

$$C = \frac{\frac{E}{x}}{R + \frac{B}{x^2}} = \frac{E}{Rx + \frac{B}{x}}.$$

For C to have a maximum value the expression $Rx + \frac{B}{x}$ must have a minimum value.

Let $Rx + \frac{B}{x} = a$; then solving the quadratic we get

$$x = \frac{a \pm \sqrt{a^2 - 4BR}}{2R},$$

and the minimum value of a which will satisfy this equation is given by

$$a^2 = 4BR, \text{ and in that case } x = \sqrt{\frac{B}{R}};$$

so that the cells of the battery must be grouped in such a way that the resultant resistance of the battery is equal to the extra-polar resistance.

In the present case $B = 24 \times 2 = 48$; $R = 4$.

$$\therefore x = \sqrt{12} = 3{\cdot}464;$$

and the integer nearest 3·464, which is also a submultiple of 24, is 3, and therefore the best way of grouping the cells will be in 3 files of 8 cells each.

(2.) How would you arrange 10 equal and similar cells, each having a resistance of 2 ohms, so as to give the strongest current through an extrapolar resistance of 2 ohms? *Answer.* All in one single file.

(3.) You have 20 exactly equal cells, each of which has an electromotive force of 1·12 volts and a resistance of 8 ohms. Show how to group them so as to obtain the maximum current through an extrapolar resistance of (1) 8 ohms; (2) of 40 ohms; (3) of 104 ohms.

Answer. (1) 5 files of 4 each; (2) 2 files of 10 each; (3) all in series.

(4.) A set of 36 exactly similar and equal cells had the ratio of the resistance of each cell to the extrapolar resistance as 4 : 1; what would be the best way of grouping the cells? *Answer.* 12 files of 3 each.

(5.) In what way must 12 Bunsen cells be joined up together so as to give the maximum current through an extrapolar resistance of 15 ohms, the electromotive force of each cell being 1·86 volts, and its resistance 1·5 ohms?

Answer. In single file.

(6.) In what way should they be arranged if the extrapolar resistance were 2 ohms?

Answer. 3 files of 4 each.

(7.) A galvanic battery was composed of a series of 20 equal cells, the electromotive force of each being 2 volts, and its resistance 3·7 ohms; how many cells of another kind should we have to arrange in series so as to form a battery having the same electromotive force between its ends, if each of the new cells has an electromotive force of 1 volt and a resistance of 6 ohms? *Answer.* 40 cells.

(8.) You have 40 Grove's cells, each of which has an electromotive force of 1·97 volts and an internal resistance of ·25 ohm. How will you arrange them for three separate experiments, (1) to work a set of Ampère's wires whose resistance is 2 ohms; (2) to work a little electromagnetic engine whose resistance is 8 ohms; and (3) to make a wire white hot whose resistance when hot is 40 ohms?

Answer. (1) 2 files of 20 each; (2) and (3) all in single file.

(9.) A sheet of zinc, one square metre in area, is to be cut up into a number of equal plates, so as to form a battery. The resistance of a cell whose plates are one square decimetre in area is 1 ohm and the extrapolar resistance is 25 ohms. What will be the best number of cells to make so as to obtain the maximum current through the extrapolar wire?

Let x = number of cells; then $\frac{100}{x}$ = area of each plate in square decimetres, and $\frac{x}{100}$ ohm is the resistance of each cell. Hence $B = \frac{x^2}{100}$ and $R = 25$.

Also, the maximum current being obtained when the battery resistance is equal to the extrapolar resistance, and since the resistance of each cell must be much less than the extrapolar resistance, it is obvious that we must have all the cells in single file;

$$\therefore\ 1 = \sqrt{\frac{x^2}{100} \times \frac{1}{25}}\,; \text{ whence } x = 50.$$

(10.) The extrapolar resistance of a certain circuit is 100 ohms; the resistance of a single cell, the area of whose zinc plate is one square decimetre is 4 ohms, and you have a piece of zinc ·9 of a square metre in area. How many separate cells should be made out of this so as to send the strongest possible current through the extrapolar resistance?

Answer. 47 cells.

(11.) The resistance of each cell of a battery of 24 cells is 1·04 ohm, and the electromotive force of the battery, when all the cells are in single file, is 10 volts, and the extrapolar resistance is 4 ohms. Find the strength of the maximum current which can be sent through this extrapolar resistance by a suitable grouping of the cells.

Answer. ·5 ampère, nearly.

(12.) A battery of 20 Daniell's cells, each of which has an electromotive force of 1 volt and a resistance of 4 ohms, is to be used for two lecture experiments, in one of which the resistance of the apparatus and leading wires is 4 ohms and in the other 40 ohms. Find the maximum current which can be sent through them in each case.

Answer. (1) $\frac{5}{9}$ ampère; (2) $\frac{1}{6}$ ampère.

(13.) What is the strongest current which can be sent for a short time through an electromagnet of 15 ohms resistance, by a battery of 100 Daniell's cells, each of which has a resistance of 2·5 ohms and an electromotive force of 1 volt? *Answer.* 816 milliampères.

(14.) Twenty-four Bunsen's cells are to be used for exciting an electromagnet whose resistance is ·25 of an ohm. The resistance of each cell is ·3 of an ohm and its electromotive force is 1·86 volt. What will be the maximum current through this electromagnet?

Answer. 16·53 ampères.

(15.) A certain incandescent lamp has a resistance, when hot, of 20 ohms and requires a current of ·9 ampère. Can it be worked by suitably grouping 50 Daniell's cells, the resistance of each being 2 ohms and its electromotive force 1 volt?

If possible let x = number of files, and y = the number of cells in each file.

Then the resistance of the battery $= \frac{2y}{x}$ ohms,

the E.M.F. of the battery $= y$ volts;

$$\therefore \mathrm{C} = \frac{y}{\frac{2y}{x} + 20} = \cdot 9 \text{ by the question.}$$

And $$xy = 50;\ \therefore y = \frac{50}{x}.$$

∴ substituting for y in terms of x we get

$$50 = \frac{1\cdot 8 \times 50}{x} + 18x;$$

whence

$$x^2 - \frac{25}{9}x + 5 = 0$$

$$\therefore x = \frac{25 \pm \sqrt{-995}}{18},$$

which involves the *even* root of a negative quantity, and therefore no arrangement of these cells will give the required current through the lamp.

VOLTAMETERS AND ELECTROLYSIS.

(1.) If one *coulomb* of electricity will decompose $9{\cdot}4687 \times 10^{-5}$ of a gramme of water, what will be the strength of a current which can electrolyse 75 milligrammes of water in 15 minutes?

Let x = strength of current in ampères. Then, since a current of one ampère furnishes one coulomb of electricity per second, the mass of water electrolysed by this current in 15 minutes will be

$$x \times 15 \times 60 \times 9{\cdot}4687 \times 10^{-2} = 75 \text{ by the question;}$$

$$\therefore x = \frac{7500}{15 \times 60 \times 9{\cdot}4687} = {\cdot}88 \text{ ampère.}$$

(2.) What will be the strength of current which in one minute develops one cubic centimetre of explosive gas in a water voltameter if a current of one ampère develops ·1764 cubic centimetre per second?

Answer. 94·48 milliampères.

(3.) In measuring the strength of a voltaic current by means of a voltameter, the following observations were made.

In 8 minutes 17·55 cubic centimetres of hydrogen were liberated. The gas was collected over dilute sulphuric acid of specific gravity 1·25 at the temperature of observation, and was assumed to be saturated with aqueous vapour.

Height of the sulphuric acid column = 33 millimetres.
Reduced height of the barometer = 757 „
Temperature of the room = 18° C.
Specific gravity of mercury = 13·596.

Calculate from these data the strength of the current.

$$\left.\begin{array}{l}\text{The volume of gas liberated}\\ \quad\text{per second}\end{array}\right\} = \frac{17{\cdot}55}{8 \times 60} \text{ c.c.}$$

$$\left.\begin{array}{l}\text{Height of sulphuric acid}\\ \quad\text{column in the tube re-}\\ \quad\text{duced to its equivalent in}\\ \quad\text{mercury}\end{array}\right\} = \frac{33\times 1{\cdot}25}{13{\cdot}596} = \left\{\begin{array}{l}\text{3 milli-}\\ \quad\text{metres,}\\ \quad\text{nearly.}\end{array}\right.$$

$$\left.\begin{array}{l}\text{Tension of aqueous vapour}\\ \quad\text{at } 18^\circ\end{array}\right\} = \left\{\begin{array}{l}15{\cdot}4 \text{ millimetres of}\\ \quad\text{mercury.}\end{array}\right.$$

$\therefore$ pressure under which the gas was collected

$$= 757 - 3 - 15{\cdot}4 = 738{\cdot}6 \text{ millimetres of mercury.}$$

$\therefore$ volume of hydrogen reduced to 0° C. and 760 millimetres pressure

$$= \frac{17{\cdot}55}{8\times 60} \times \frac{738{\cdot}6}{760} \times \frac{273}{291} = {\cdot}033335 \text{ c.c.}$$

But one ampère gives one coulomb per second and liberates ·1176 cc. of hydrogen per second; $\therefore$ if x represent the strength of the current in ampères,

$$x = \frac{{\cdot}033335}{{\cdot}1176} = {\cdot}28346 \text{ ampère}$$

$$= 283{\cdot}5 \text{ milliampères, nearly.}$$

(4.) In a similar experiment the current was sent for 10 minutes through a voltameter, and the hydrogen was collected over dilute sulphuric acid of specific gravity 1·25. The volume of damp hydrogen evolved in 10 minutes was 22·3 cubic centimetres, and the height of the column of sulphuric acid in the tube at the end of the experiment was 42 millimetres. The temperature of the room was 13° C., and the observed height of the barometer 754·3 millimetres. Taking the specific gravity of mercury as 13·6, find the strength of the current in milliampères.

Answer. 293·3 milliampères.

(5.) A tangent galvanometer and a water voltameter were included in the same circuit, and it was observed that a deflection of 30° was produced in the galvanometer while at the same time a volume of 49·7 cubic centimetres of explosive gas was developed per minute. How many cubic centimetres of mixed gas would be developed per minute by a current which deflected the needle through an angle of 45°?

Since the current strength is directly proportional to the quantity of gas developed per second and to the tangent of the angular deflection of the needle, we must have

$$\frac{x}{49{\cdot}7} = \frac{\tan 45^\circ}{\tan 30^\circ}, \therefore x = 49{\cdot}7 \times \sqrt{3} = 86{\cdot}08 \text{ c.c.}$$

Let C represent the strength of the current in ampères, then a deflection of 45° would correspond to a development of $\frac{86{\cdot}08}{60}$ c.c. of gas per second. But one coulomb of electricity developes ·1764 c.c. of mixed gas,

$$\therefore \text{C} = \frac{86{\cdot}08}{60 \times {\cdot}1764} = 8{\cdot}133 \text{ ampères.}$$

Also from the formula

$$\frac{\text{C}_2}{\text{C}_1} = \frac{\tan \theta_2}{\tan \theta_1}, \text{ on putting } \theta_1 = 45^\circ \text{ and } \text{C}_1 = 8{\cdot}133,$$
$$\text{C}_2 = 8{\cdot}133 \times \tan \theta_2;$$

that is to say that, if we use the galvanometer in the same position, the absolute value of any current in ampères will be obtained by multiplying the tangent of the angular deflection of the needle by 8·133. The factor 8·133 is often called the galvanometer 'constant.'

(6.) How many cubic centimetres of mixed gas would have been developed per minute by a current of such a strength that the deflection of the needle was 43° 25′?

Answer. 81·452.

(7.) Find the numerical value of the *constant* of a certain tangent galvanometer in which the deflection of the needle was 37° 27′ while the current evolved 35 cubic centimetres of mixed gas per minute. *Answer.* 4·317.

(8.) In an experiment to determine by means of a voltameter the *constant* of a tangent galvanometer, it was observed that 53·5 cubic centimetres of dry explosive gas were collected per minute while the deflection of the needle was 38° 29′, the laboratory barometer standing at 758·9 millimetres and the thermometer at 15° C. Find the *constant* of this galvanometer at 0° C. and 760 millimetres of barometric pressure. *Answer.* 1·062, nearly.

(9.) A tangent galvanometer and a water voltameter were joined up with a battery, and a deflection of 22° was obtained while the quantity of explosive gas evolved per minute was 30·8 cubic centimetres, the temperature of the room being 15° C. and the barometric column 740 millimetres. What would be the quantity of explosive gas reduced to 0° C. and 760 millimetres pressure which would be evolved per minute by a current which produced a deflection of 45°? *Answer.* 70·36.

(10.) If with another current the mean angular deflection of the needle was 46° 23′, what quantity of explosive gas would this current evolve per minute from the water in a voltameter? *Answer.* 73·84 c.c., nearly.

(11.) A tangent galvanometer, whose *constant* had previously been determined and found equal to ·876, was joined up in series with another galvanometer and a battery, and the simultaneous deflections of the two instruments were 68° and 32°. Find the constant of the galvanometer whose deflection was 32°. *Answer.* 3·47, nearly.

(12.) The same current was sent simultaneously through two voltameters, one of them containing acidulated water and the other a solution of sulphate of copper. In the course of one minute there was evolved a volume of explosive gas which, corrected for temperature and pressure, was 60 cubic centimetres. What was the weight of copper deposited per minute in the other voltameter?

The number of coulombs of electricity corresponding to 60 c.c. of mixed gas

$$= \frac{60}{\cdot 1764} = 340\cdot 1.$$

The atomic weight of copper, which is a dyad element, being 63·5, its chemical equivalent, which is

$$\frac{\text{atomic weight}}{\text{valency}}, \text{ is } \frac{63\cdot5}{2} = 31\cdot7.$$

But one coulomb of electricity liberates $1\cdot05 \times 10^{-4}$ gramme of hydrogen, and ∴ $31\cdot7 \times 1\cdot05 \times 10^{-4}$ gramme of copper. Hence the mass of copper liberated by 340·1 coulombs will be $340\cdot1 \times 31\cdot7 \times 1\cdot05 \times 10^{-1} = 113\cdot2$ milligrammes.

(13.) Two voltameters were included in the same electric circuit, one of them containing a solution of sulphate of copper, and the other a solution of nitrate of silver. It was found that after a certain time 117·2 milligrammes of copper had been deposited in one voltameter. How much silver was deposited in the other voltameter in the same space of time? *Answer.* 399·3 milligrammes, nearly.

(14.) On another occasion, and with a current of different strength, it was found that at the end of an hour and a half 878·1 milligrammes of silver had been deposited. How much copper had been deposited in the other voltameter in the same interval of time?

Answer. 257·7 milligrammes.

(15.) Three voltameters, one containing acidulated water, another containing fused oxide of lead, and the third, fused chloride of tin, were joined up in the same simple circuit with a powerful battery. It was found that the mass of water electrolysed in a certain time was 118·7 milligrammes. How much lead and how much tin would be deposited in the same time, in their respective cells?

Answer. $\left.\begin{array}{l}1362\cdot1 \text{ milligrammes of lead}\\ 766\cdot5 \quad ,, \qquad ,, \quad \text{tin}\end{array}\right\}.$

(16.) In an electrosilvering apparatus the resistance of the liquid between the electrodes and of the connecting wires is ·2 ohm. Forty cells are to be employed, each having an electromotive force of 1·97 volts and an internal resistance of ·4 ohm. What will be the best way of grouping these cells so as to deposit the silver most rapidly, and in that case find the resulting current, the amount of silver deposited, and the whole amount of zinc dissolved per minute?

By the method of Example 1 of the previous section we have $x = \sqrt{\frac{16}{\cdot 2}} = 9$, nearly, and therefore the best way of grouping the cells will be in 10 files of 4 cells each.

In that case $\Sigma\,\text{E} = 4 \times 1{\cdot}97 = 7{\cdot}88$ volts.

$$\Sigma\,\text{R} = {\cdot}2 + {\cdot}16 = {\cdot}36 \text{ ohm};$$

$$\therefore\ \text{C} = \frac{7{\cdot}88}{{\cdot}36} = 21{\cdot}889 \text{ ampères.}$$

$\therefore$ Silver deposited per minute $= 60 \times 21{\cdot}889 \times 1{\cdot}134 = 1489{\cdot}3$ milligrammes, since one coulomb liberates ·001134 gramme of silver.

Again, the battery being arranged in 10 files of 4 cells each, it follows that in each *row* of 10 cells there will be a quantity of zinc dissolved per minute which is equal to

$$60 \times 21{\cdot}889 \times {\cdot}3412 = 448{\cdot}17 \text{ milligrammes.}$$

$\therefore$ The amount of zinc dissolved in the whole battery in a minute $= 448{\cdot}17 \times 4 = 1792{\cdot}68$ milligrammes.

Hence for every gramme of zinc dissolved we get

$$\frac{14893}{17926{\cdot}8} = {\cdot}8308 \text{ gramme of silver.}$$

(17.) Suppose all the cells were placed abreast, and their like poles connected together by thick wires and then with the electrolytic vessel. What amount of silver would be deposited per minute, and what would be the expenditure of zinc in the same time?

Silver deposited per minute = 641·5 milligrammes
Zinc dissolved ,, = 193 ,,

$\therefore$ For every gramme of zinc we get $\frac{{\cdot}6415}{{\cdot}193} = 3{\cdot}3238$ grammes of silver.

N.B. The student will notice that the deposition by the slower method is much more economical of zinc. In fact, nearly in the ratio 4 : 1.

(18.) The mould for a copper medal was placed in an electrotyping apparatus and the total resistance external to the battery was ·005 ohm. The battery consisted of a single large Daniell's cell, whose resistance was $\frac{1}{10}$ ohm and its electromotive force 1·12 volt. How much copper would be deposited in an hour on the mould, and how much zinc would be dissolved in the cell in the same time?

Answer. $\left.\begin{array}{l}\text{Copper} = 12{\cdot}7 \text{ grammes} \\ \text{Zinc} \quad = 13{\cdot}1 \quad \text{,,}\end{array}\right\}$, approximately.

(19.) An electric current was sent for 24 hours through an electrolytic cell containing a solution of sulphate of copper, and at the end of the time 2·3 grammes of copper had been deposited. Had the cell contained acidulated water instead of copper sulphate, what mass of hydrogen would have been liberated in the same time?

Answer. 73 milligrammes.

(20.) What must be the average strength of a current circulating through a solution of silver nitrate, so as to deposit 144 grammes of silver in 24 hours?

Answer. 1·47 ampère, nearly.

(21.) If a battery of 20 Daniell's cells in series send, for 24 hours, a current through an extrapolar resistance of 600 ohms, how much zinc will have been dissolved in each cell of the battery during that time, taking the average resistance of each cell at 5 ohms and its electromotive force at 1·12 volt? *Answer.* 943·4 milligrammes, nearly.

(22.) A voltameter containing acidulated water was included in circuit with a single file of 5 of Grove's cells. It was observed that 41·76 cubic centimetres of explosive gas were given off per minute. How many milligrammes of zinc were dissolved per minute in each cell of this battery?

Answer. 80·8 milligrammes.

(23.) If the current from a battery of 10 Grove's cells in series is sent simultaneously through two voltameters in series, one of them containing a solution of copper sulphate and the other a solution of silver nitrate, find how much silver and how much copper will be deposited while 3·25 grammes of zinc are dissolved in the entire battery. Also find how much copper and silver would be deposited during the solution of the same quantity of zinc in the entire battery, if it were arranged in two files of 5 cells each instead of a single file of 10 cells.

$$\textit{Answer.}\quad (1)\ \left.\begin{array}{l}\text{Silver} = 1\cdot08 \text{ gramme}\\ \text{Copper} = \cdot317 \quad ,,\end{array}\right\};\qquad (2)\ \left.\begin{array}{l}\text{Silver} = 2\cdot16 \text{ grammes}\\ \text{Copper} = \cdot63 \quad ,,\end{array}\right\}.$$

(24.) The current from a battery of 70 Daniell's cells in series was sent, for 24 hours, through an extrapolar resistance of 3200 ohms. Taking the electromotive force of each cell at 1·12 volt, and its resistance equal to 8 ohms, what was the whole amount of zinc consumed in the battery during that time? *Answer.* 43·028 grammes.

(25.) Suppose that instead of the Daniell's cells 70 Grove's cells had been used, each having an electromotive force of 1·8 volt and a resistance of ·9 ohm, how much zinc would have been consumed in the 24 hours?

Answer. 79·685 grammes.

(26.) A tangent galvanometer and a water voltameter were included in the same voltaic circuit, and it was found that with a current which produced a deflection of 41° the quantity of explosive gas developed per minute was 74·5 cubic centimetres. If this same tangent galvanometer, used in the same position, was included in a circuit in which the current was made to pass through a voltameter containing sulphate of copper solution, and a current whose average strength would be represented by a deflection of the needle of 10° was sent through it for 24 hours, how much copper would be deposited in that time?

From the first part of the problem we find that the constant of the galvanometer is 88·702.

∴ A current producing a deflection of 10° would liberate a quantity of explosive gas $= 88·702 \times \tan 10° = 15·116$ c.c. per minute.

$$\therefore \text{ Strength of current} = \frac{15·116}{60 \times ·1764} = 1·428 \text{ ampère.}$$

∴ Amount of copper deposited in 24 hours

$= 1·428 \times 3600 \times 24 \times ·0003307 = 40·807$ grammes.

(27.) A certain Saxton's magneto-electric machine, when kept in continuous revolution, was able to precipitate 2·5 kilogrammes of silver per week from its solution. What was the average strength of current?

Answer. 7·5819 ampères.

(28.) Another of these machines was able to deposit ·25 kilogramme of silver per hour upon articles properly prepared for this mode of plating. Find the strength of the current. *Answer.* 127·4 ampères.

CHEMICAL THEORY OF ELECTROMOTIVE FORCE.

(1.) Supposing that the effective action in a Grove's cell is due to the oxidation of the zinc and the reduction of the nitric acid to nitrous acid; that the heat evolved by the oxidation of one equivalent of zinc is 53258 gramme-units, while the heat absorbed in the reduction of one equivalent of nitric acid to nitrous acid is 13634 units; find the electromotive force of a Grove's cell.

The chemical equivalent of zinc being 32·5, the quantity of zinc deposited by the C.G.S. electromagnetic unit of electricity (10 coulombs) is ·003412 gramme. Hence, while

32·5 grammes of zinc are dissolved, the quantity of electricity developed

$$= \frac{32{\cdot}5}{{\cdot}003412} = 9525{\cdot}2 \text{ C.G.S. electromagnetic units.}$$

But the effective heat liberated during this process is 53258 − 13634 = 39624 gramme-units, and if we multiply this by Joule's equivalent, $4{\cdot}2 \times 10^7$, we get $39624 \times 4{\cdot}2 \times 10^7$ ergs for the chemical work done in the cell while one equivalent of zinc is being dissolved. But the electromotive force of an electrochemical apparatus is in absolute measure equal to the mechanical equivalent of the chemical action on one electrochemical equivalent of the substance; and in the present case the work done on each electromagnetic unit of electricity is

$$= \frac{39624 \times 4{\cdot}2 \times 10^7}{9525{\cdot}2} = 1{\cdot}747 \times 10^8$$
$$= 1{\cdot}747 \text{ volt,}$$

since 1 volt = 10^8 C.G.S. electromagnetic units of potential.

(2.) If we assume that the oxidation of one gramme of zinc evolves 1633·7 gramme-units of heat, and that the reduction of one gramme of copper absorbs 933·91 units, also that the effective chemical action which produces the current consists in the oxidation of the zinc and the reduction of the copper, calculate the electromotive force of a Daniell's cell. *Answer.* 1·044 volt.

(3.) Favre and Silbermann have found that one gramme of zinc in replacing the copper in copper sulphate disengages 714 gramme-units of heat. Calculate from this the absolute electromotive force of a Daniell's cell.

By the method adopted in Example 1 we shall find that the work done upon one C.G.S. electromagnetic unit of electricity is

$$= 714 \times 4{\cdot}2 \times 10^7 \times {\cdot}003412,$$

$$\therefore \text{ E.M.F. in volts} = \frac{714 \times 4{\cdot}2 \times 10^7 \times {\cdot}003412}{10^8}$$
$$= 1{\cdot}023 \text{ volts.}$$

(4.) The same two observers have found that one gramme of zinc in replacing the hydrogen in sulphuric acid disengages 567·9 heat-units. Deduce from this the absolute electromotive force of a Smee's cell.

Answer. ·814 volt, nearly.

(5.) According to Regnault the electrolysis of one equivalent of water absorbs 34462 heat-units, while the solution of one equivalent of amalgamated zinc in dilute sulphuric acid evolves 53258 units. Find the electromotive force of a Wollaston's cell on the assumption that it is proportional to the amount of heat liberated.

Answer. ·829 volt.

(6.) Find the electromotive force of a Daniell's cell from the following data, *vide* 'Phil. Mag.', 1851 :

Heat evolved by the combination of 1 gramme of zinc with oxygen = 1301 units.

Heat evolved by the combination of one gramme of oxide of zinc with dilute sulphuric acid = 296·148 „

Heat evolved by the combination of one gramme of copper with oxygen = 605·12 „

Heat evolved by the combination of one gramme of oxide of copper and dilute sulphuric acid = 239·967 „

Electrochemical equivalent of zinc = ·00342

Joule's coefficient = 41572500

Solution.

One gramme of zinc corresponds to $\frac{81}{65}$ = 1·246 gramme oxide of zinc.

One gramme of zinc corresponds to $\frac{63}{65}$ = ·969 gramme oxide of copper.

·969 gramme of copper corresponds to $\frac{79}{63} \times$ ·969 = 1·2151 gramme oxide of copper.

∴ Heat *evolved* by combination of one gramme of zinc with oxygen	= 1301	units.
Heat *evolved* by combination of 1·246 gramme zinc oxide with dilute sulphuric acid	= 369	„
Total	= 1670	„

Heat *absorbed* by separation of ·969 gramme of copper from its oxide = ·969 × 605·12	= 586·36	
Heat *absorbed* by separation of 1·2151 gramme of oxide of copper	= 291·58	
Total	= 877·94	units.

∴ Total heat evolved for one gramme of zinc dissolved

$$= 1670 - 877{\cdot}94 = 792{\cdot}06 \text{ units.}$$

∴ Total work done for one electromagnetic unit of electricity is

$$792{\cdot}06 \times {\cdot}00342 = 2{\cdot}7088,$$

and ∴ the chemical work done for one C.G.S. unit of electricity is

$$= 2{\cdot}7088 \times 41572500 \text{ ergs};$$

$$\therefore \text{ electromotive force} = \frac{2{\cdot}7088 \times 41572500}{10^8}$$

$$= 1{\cdot}126 \text{ volt.}$$

(7.) Find the electromotive force of a Wollaston's cell from the following data :

Heat evolved by combination of 1 gramme of zinc with oxygen	= 1301	units.
Heat evolved by combination of 1 gramme of hydrogen with oxygen	= 34462	„

Heat evolved by combination of 1 gramme of anhydrous oxide of zinc and sulphuric acid } = 270 units.

Electrochemical equivalent of zinc } = ·00342

Solution.

Heat evolved by combination of 1 gramme of zinc with oxygen } = 1301 units.

Heat absorbed in separating $\frac{1}{32·5}$ gramme of hydrogen from water $= \frac{34462}{32·5} = 1060·368$ } = 1060·368

Heat evolved by the combination of $\frac{81}{65}$ grammes of anhydrous oxide of zinc with dilute sulphuric acid $= \frac{81}{65} \times 270$ } = 336·46

Balance of heat evolved = 577·09 units.

Hence, as in the solution of the last example,

$$\text{E.M.F. of cell} = \frac{577·09 \times ·00342 \times 41572500}{10^8}$$

$$= ·8205 \text{ volt.}$$

(8.) Taking the chemical equivalent of zinc as 32·5, and assuming that the combination of one gramme of zinc with oxygen evolves 1301 units of heat, and that the burning of one gramme of hydrogen in oxygen evolves 34462 units, find the electromotive force of a Smee's cell on the hypothesis of Faraday that the effective chemical action in producing the current consists in the oxidation of the zinc and the deoxidation of the hydrogen.

Answer. ·342 volt, nearly.

(9.) Assuming that the electrochemical equivalent of hydrogen is ·0001, and that the heat developed in the combination of one gramme of hydrogen with oxygen is 34462 units, find the least electromotive force that would be required to decompose water.

Answer. Taking $J = 4{\cdot}2 \times 10^7$, the minimum E.M.F. = 1·447 volt.

N.B.—On comparing the results of the previous examples, the student will notice that, according to these results, 2 Daniell's cells at least would be required to effect the electrolysis of water, and that Grove's is the only kind of voltaic cell by which it is possible to do this with only *one* cell.

THERMO-ELECTRICITY

(1.) A single copper-iron junction was placed in melting ice, the other junctions and the rest of the circuit being at the temperature of the room, i.e. 16° C. The rest of the circuit consisted of a coil wire of 20 ohms resistance and a galvanometer of 6 ohms resistance. What was the strength of current?

The thermo-electric power of a copper-iron junction, when the temperature of the junction is t° C. above or below that of the rest of the circuit, is $13{\cdot}7 \times t$ microvolts.

In the present case $E = 13{\cdot}7 \times 16 = 219{\cdot}2$ microvolts,

$R = 20 + 6 = 26$ ohms;

$$\therefore\ C = \frac{219{\cdot}2 \times 10^{-6}}{26 \times 10^{3}} \text{ ampère, or } {\cdot}0084 \text{ milliampère.}$$

(2.) In another experiment with the same thermo-electric arrangement, the temperature of the room was 13° C., and the extrapolar resistance consisted of a galvanometer of ·25 ohm and a wire of 5 ohms resistance. What was the current strength? *Answer.* ·034 milliampère, nearly.

(3.) One junction of a thermo-electric couple of bismuth and antimony was kept in boiling water, while the temperature of the rest of the circuit was 0° C. The extrapolar circuit consisted of a wire of 4 ohms resistance and a galvanometer of $\frac{1}{2}$ an ohm. What was the current strength, the thermo-electric power of the couple per degree Centigrade being 117 microvolts? *Answer.* 2·6 milliampères.

(4.) A thermo-electric pile consists of 96 pairs of bismuth and antimony bars, soldered together in the usual manner. What will be the electromotive force produced by a difference of ·05° C. in the temperatures of the opposite faces?

Answer. 561·6 microvolts.

(5.) Find the electromotive force of a thermo-electric battery consisting of 200 platinum and iron couples, the alternate junctions being kept at the temperature of 0° C. and 100° C., having given that with respect to lead;

thermo-electric value of iron is $-1734 + 4{\cdot}87\,t$,
" " " platinum is $-\ 260 + {\cdot}75\,t$.

The thermo-electric value for platinum-iron is obtained by subtracting that of iron with lead from that of platinum with lead, in the above expression, and is therefore

$$1474 - 4{\cdot}12\,t.$$

The mean of the temperatures of the junctions is 50° C., and ∴ the electromotive force per degree Centigrade for one couple is

$1474 - 4{\cdot}12 \times 50 = 1268$ electromagnetic units of potential.

∴ Electromotive force of this battery

$$= \frac{1268 \times 200 \times 100}{10^8} \text{ volts} = {\cdot}2536 \text{ volt.}$$

(6.) A rod of lead was bent, as in Pouillet's experiments, into the shape of three sides of a rectangle, and a long copper wire was soldered to the ends which were placed, one in melting ice, and the other in water at 15° C. What would be the electromotive force in the circuit, assuming that, according to Matthiessen's experiments, the thermo-electric value of iron with respect to lead is

$$-136-{\cdot}95\,t \text{ electromagnetic units,}$$

where t is the mean of the temperatures of the junctions.

In the present case $t = 7^{\circ}{\cdot}5$ C., ∴ thermo-electric force in electromagnetic units for 1° C. difference of temperature of the junctions is

$$-136-{\cdot}95 \times 7{\cdot}5 = -143{\cdot}12.$$

∴ E.M.F. in circuit

$$= \frac{143{\cdot}12 \times 15}{10^8} \text{ volts} = 21{\cdot}468 \text{ microvolts.}$$

(7.) Find the electromotive force of a zinc-copper couple, the temperatures being 0° C. and 100° C., having given that with respect to lead as zero, in electromagnetic units,

thermo-electric value for zinc is $-234-2{\cdot}40\,t$,
" " " copper is $-136-{\cdot}95\,t$.

Answer. 170·5 microvolts.

(8.) What would be the electro-motive force of the same couple if the temperatures of the junctions were 15° and 75° C. respectively? *Answer.* 97·95 microvolts.

(9.) A bar of aluminium is bent into the form of three sides of a rectangle, and to the ends there is soldered a long piece of German silver wire, and the temperatures of the junctions are kept at 13° C. and 37° C. respectively. What would be the electromotive force, having given that with respect to lead;

thermo-electric value of German silver is $+1207+5{\cdot}12\,t$,
" " " aluminium is $+77-{\cdot}39\,t$?

Answer. 304·26 microvolts.

(10.) Find the neutral point of platinum and iron from the data in Example 5.

Cumming found that the order of certain metals in the thermo-electric scale is different at high and low temperatures, so that at a particular temperature two metals may be neutral to each other. In the present case we have

$$-1734+4{\cdot}87\,t = -260 + {\cdot}75\,t,$$

whence for the *neutral point* $t = \frac{1474}{4{\cdot}12} = 357^\circ{\cdot}8$ C.

(11.) Find the neutral point for tin and iron, having given that with respect to lead as zero,

thermo-electric value for tin is $+43 - {\cdot}55\,t$.

Answer. $t = 327^\circ{\cdot}9$ C., nearly.

(12.) Find the electromotive force of a thermo-pile consisting of 769 elements of iron and German silver, the temperatures of the junctions being 13° C. and 28° C. respectively. *Answer.* ·34 volt, nearly.

BATTERY RESISTANCE.

(1.) A single galvanic cell was connected up with a tangent galvanometer and a resistance box. When the extrapolar resistance was 1·44 ohm, the deflection of the needle was 45° 30′, and when an additional resistance of one ohm was included in the circuit, the deflection was 32° 20′. Find the internal resistance of the cell.

Let E represent the electromotive force of the cell in volts,
„ B „ „ resistance „ ohms.
Then by Ohm's law

$$\left.\begin{aligned}\frac{E}{B + 1{\cdot}44} &= m \tan 45^\circ\ 30' \\ \frac{E}{B + 2{\cdot}44} &= m \tan 32^\circ\ 20'\end{aligned}\right\};$$

$$\therefore\ \frac{B+2{\cdot}44}{B+1{\cdot}44} = \frac{\tan 45^\circ\ 30'}{\tan 32^\circ\ 20'} = \frac{1{\cdot}0176}{{\cdot}6331},\ \text{whence } B = {\cdot}2 \text{ ohm.}$$

N.B. The measurement of the resistance of a battery or of a single cell when in action is always a matter of difficulty, on account of the fact that the working electromotive force is not strictly constant throughout the experiment.

(2.) A medium-sized Bunsen's cell was joined up with a tangent galvanometer of very small resistance, and produced a deflection of 33° 30′. An additional resistance of 1·39 ohm was then introduced into the circuit and the deflection fell to 8°. Find the resistance of the cell, neglecting that of the galvanometer. *Answer.* ·37 ohm, nearly.

(3.) With a much larger Bunsen's cell and the same galvanometer a deflection of 57° was obtained, and then by means of a Wheatstone's rheostat a resistance of ·145 ohm was added to the circuit, and the deflection of the needle fell to 38°. What was the resistance of this cell, neglecting that of the galvanometer? *Answer.* ·15 ohm, nearly.

(4.) In order to find the resistance of a certain battery it was joined up with a tangent galvanometer of 98·5 ohms resistance, and a wire resistance of 175 ohms, when the deflection of the needle was 37°. When the wire resistance was increased to 728 ohms, the deflection became 20°. What was the battery resistance?

Let G represent the resistance of the galvanometer in ohms.

Let R represent the resistance of the wire in ohms.

Then, as before,

$$\left.\begin{aligned}\frac{\text{E}}{\text{G}+\text{R}+\text{B}} &= m \tan 37^\circ\\ \frac{\text{E}}{\text{G}+\text{R}'+\text{B}} &= m \tan 20^\circ\end{aligned}\right\} \therefore \frac{\text{B}+826\cdot5}{\text{B}+273\cdot5} = \frac{\cdot754}{\cdot364};$$

whence $\text{B} = 242\cdot6$, nearly.

(5.) A battery whose resistance was required was joined up with a tangent galvanometer whose resistance was ·325 ohm and a standard resistance of 17 ohms, when a deflection of 45° was obtained. An additional resistance of 25·3 ohms was then included, so as to bring the deflection down to 27°. What was the resistance of the battery?

Answer. 8 ohms, nearly.

(6.) The current from a battery was sent through a tangent galvanometer of 4 ohms resistance, and produced a certain deflection of the needle. An additional resistance of 83 ohms had to be included in the circuit in order that the deflection of the needle might be such that its tangent was equal to one-half of that of the former deflection. Find the resistance of the battery. *Answer.* 79 ohms.

(7.) A galvanic cell was formed by immersing a plate of copper and a plate of zinc, each of 10 square centimetres in area, at a distance of 1·5 centimetre apart in dilute sulphuric acid whose specific resistance per cubic centimetre was 5·12 ohms. Find the ohmic resistance of this cell.

Ohmic resistance

$$= \frac{\text{distance apart of plates} \times \text{specific resistance}}{\text{area of plates}}$$

$$= \frac{1·5 \times 5·12}{10} = ·768 \text{ ohm.}$$

(8.) In another simple voltaic cell the areas of the copper and zinc plates were each 20 square centimetres, and the distance of the plates was one centimetre. The liquid between them was acidulated water whose specific resistance per cubic centimetre was 9·28 ohms. What was the ohmic resistance of the cell? *Answer.* ·464 ohm.

(9.) A battery, a reflecting galvanometer of 50 ohms resistance, and a standard resistance of 1000 ohms, were joined up together, when the spot of light was deflected over 56 divisions of the scale. In order to bring the deflection down to 28 scale-divisions, so as to reduce the current to one-half its initial value, an additional resistance of 1133 ohms had to be introduced into the circuit. What was the resistance of the battery?

In this case

$$\frac{\text{B} + \text{G} + 1000}{\text{B} + \text{G} + 2133} = \tfrac{1}{2}, \ \therefore \text{B} + \text{G} = 133, \ \therefore \text{B} = 83 \text{ ohms.}$$

Note.—The student will notice that this method involves no calculation, as we have only to deduct the original extra-polar resistance from the *added resistance* to obtain the resistance of the battery.

(10.) A voltaic battery was joined up in circuit with a resistance box and a reflecting galvanometer whose resistance was 150 ohms. With 500 ohms from the box the deflection of the spot of light was equal to 250 scale-divisions; and in order to reduce this to 125 divisions, 730 ohms additional resistance had to be introduced from the box. What was the battery resistance?

Answer. 80 ohms.

(11.) A battery, a box of resistance coils, and a reflecting galvanometer of 100 ohms resistance, were joined up in simple circuit, and when the resistance from the box was 450 ohms, the deflection of the spot of light was 27 scale-divisions, but when the resistance was 280 ohms, the deflection was equal to 36 divisions. Find the resistance of the battery.

Here $$\frac{\text{B} + 100 + 280}{\text{B} + 100 + 450} = \tfrac{27}{36} = \tfrac{3}{4}; \ \therefore \text{B} = 130 \text{ ohms.}$$

(12.) A battery of 50 cells was joined up in circuit with a reflecting galvanometer of 5150 ohms resistance and a standard resistance of 6800 ohms, and a deflection of 50 scale-divisions was obtained. On introducing an additional resistance of 1342 ohms into the circuit, this deflection was reduced to 45 divisions. Find the average resistance of each cell. *Answer.* 2·56 ohms.

(13.) A portion of the wire which formed the extrapolar circuit of a single Grove's cell was stretched horizontally at right angles to the magnetic meridian. A small magnetic needle, suspended freely just over this portion of the wire, made 33 oscillations a minute when only under the influence of the earth's magnetic force. When the extrapolar resistance was made equal to 4 ohms and 3 ohms respectively, and the current was passing, the needle made 41 and 43 oscillations in a minute. Find the resistance of the cell.

This is called Fechner's method.

Let x = resistance of the cell in ohms, then we shall have

$$\left.\begin{aligned} \frac{\mathrm{E}}{4+x} &= m(41^2 - 33^2) = m \times 592 \\ \frac{\mathrm{E}}{3+x} &= m(43^2 - 33^2) = m \times 760 \end{aligned}\right\}$$

$$\therefore \frac{4+x}{3+x} = \frac{760}{592}, \text{ whence } x = \cdot 5 \text{ ohm, nearly.}$$

(14.) In another pair of experiments the same needle performed 41 and 46 oscillations in a minute when the extrapolar resistances were 4 and 2 ohms respectively. What was the resistance of the cell employed?

Answer. ·7 ohm, nearly.

(15.) To determine the resistance of a certain battery by Sir W. Thomson's method it was joined up with a galvanometer, a box of resistance coils, and a shunt between the poles of the battery. The resistance of the galvanometer was 97·5 ohms, that of the shunt 1000 ohms, and that in the box 560 ohms; and with this a certain deflection was obtained. The shunt was then removed, and in order to bring back the original deflection of the needle, 220 ohms additional resistance had to be introduced. Find the resistance of the battery.

Let B = resistance of the battery in ohms.
G = " " galvanometer "
S = " " shunt "
R = " from the box "
r = " added "

The current flowing through the battery when the shunt is *in* is

$$= \frac{E}{\frac{(G+R)S}{G+R+S} + B} = \frac{E(G+R+S)}{(G+R)S + B(G+R+S)},$$

and the current through galvanometer

$$= \frac{S}{G+S+R} \times \text{current through battery}$$

$$= \frac{S \times E}{(G+R)S + B(G+R+S)}.$$

When the shunt is removed, and the additional resistance (r) is inserted, the current through the galvanometer is

$$= \frac{E}{G+R+r+B},$$

and since, by hypothesis, these currents are of equal strength we have

$$\frac{SE}{(G+R)S + B(G+R+S)} = \frac{E}{G+R+r+B},$$

whence $$B = \frac{Sr}{G+R} = \frac{1000 \times 220}{657{\cdot}5} = 334{\cdot}6 \text{ ohms.}$$

(16.) A single cell was joined up, in the manner described in the last Example, with a galvanometer of 3·7 ohms resistance, a standard resistance of 6·3 ohms, and a shunt across the poles of the cell of 10 ohms. On removing the shunt 2·3 ohms had to be added to the extrapolar circuit so as to reproduce the original deflection. What was the resistance of the cell? *Answer.* 2·3 ohms.

(17.) A galvanometer, whose resistance was 96 ohms, was joined up with a standard resistance of 540 ohms, the battery whose resistance was required and a shunt across the battery terminals of 100 ohms and the deflection of the needle was noted. On removing the shunt an additional resistance of 312 ohms had to be included in the circuit so as to bring the needle back to its original position. Find the resistance of the battery.

Answer. 49 ohms, nearly.

(18.) The poles of a battery of 50 cells are connected by a shunt of 140 ohms resistance. The positive pole of the battery is also connected with the positive pole of a battery of 40 cells, similar to the first, the negative pole of which is connected with the negative pole of the first battery through a galvanometer. If, then, there is no current flowing through the galvanometer, find the resistance of the first battery.

This is really Poggendorff's method, and the student should consult the solution of Example 12 in 'Currents in Branch Circuits.'

Let B = resistance in ohms of battery B (see fig. 11).
S = " " shunt.
n = number of cells in battery B.
n' = " " B$'$.
e = electromotive force of one cell of either B or B$'$.

Now since the difference of potential between C and D due to the battery B just balances the electromotive force of the battery B$'$, it must $= n'e$, $\therefore$ current in shunt $= \dfrac{n'e}{\text{S}}$.

But current in CBD

$$= \frac{\text{total electromotive force in circuit}}{\text{resistance of battery B}} = \frac{ne - n'e}{\text{B}},$$

and since no current flows through the galvanometer, the current in the shunt must be equal to the current through the battery B,

$$\therefore \frac{n'e}{\text{S}} = \frac{ne - n'e}{\text{B}};$$

$$\text{whence B} = \text{S}\left(\frac{n}{n'} - 1\right) = 140\left(\frac{50}{40} - 1\right)$$
$$= 35 \text{ ohms.}$$

(19.) A battery of 5 Daniell's cells was joined up to another battery of 3 Daniell's cells and a galvanometer in the manner described in the last Example. Across the poles of the first battery an adjustable shunt was placed, and the shunt had to be made equal to 18 ohms, so that no current should flow through the galvanometer. What was the average resistance of one of these cells?

Answer. 2·4 ohms.

(20.) In order to determine, by Mance's method, the resistance of a large Leclanché cell, it was placed in the arm B C of a Wheatstone bridge A B C D (see fig. 12), a contact breaker was placed in B D, and A and C were connected by a galvanometer. The resistances of the arms A B and A D were 6 and 15 ohms respectively, and when D C was made 10 ohms the deflection of the galvanometer remained constant, whether contact was made or broken in B D. Find the resistance of the cell.

If the resistances of the branches of the bridge satisfy the condition $R_1 R_4 = R_3 R_2$, then B D is conjugate to C A, and in that case, making or breaking contact in B D will produce no change in the current flowing through the galvanometer G.

In the present case we have

$$R_3 = \frac{R_1 R_4}{R_2} = \frac{6 \times 10}{15} = 4 \text{ ohms.}$$

N.B. The objections to Mance's method are fully described by Professor O. Lodge, 'Proceedings of the Physical Society,' vol. ii. part iii. p. 147.

(21.) A Daniell's cell was placed in the branch B C of the bridge described in the last Example. The fixed resistances A B and A D were made 5 and 10 ohms, and when the branch D C was made 15 ohms the current in the galvanometre was unaltered on making or breaking contact in B D. Find the resistance of this cell. *Answer.* 7·5 ohms.

ELECTROMOTIVE FORCE.

(1.) A Daniell's and a Grove's cell were connected up in simple circuit with a tangent galvanometer, and when their electromotive forces acted in the same direction, the deflection of the needle was 42°, but when the position of the Daniell was reversed, the deflection was 14°. Compare their electromotive forces.

Let E represent the electromotive force of the Grove cell.
E′ „ „ „ „ Daniell „
R „ resistance of the whole circuit.

Then by Ohm's law

$$\left.\begin{aligned}\frac{E+E'}{R}&=m\tan 42^\circ\\ \frac{E-E'}{R}&=m\tan 14^\circ\end{aligned}\right\}\therefore\ \frac{E+E'}{E-E'}=\frac{\cdot 9004}{\cdot 2493};$$

$$\therefore\ \frac{E}{E'}=\frac{1\cdot 1497}{\cdot 6511}$$

$$=1\cdot 77 \text{ nearly.}$$

(2.) Two batteries, A and B, were joined up in series so that their currents went in the same direction through a large resistance and a reflecting galvanometer, and the observed deflection of the spot of light was 483 scale-divisions. On reversing the battery B a deflection of 45 divisions was obtained. Compare the electromotive forces of these two batteries. *Answer.* A : B :: 121 : 100, nearly.

(3.) A similar experiment was performed with two other batteries, and it was found that when their electromotive forces acted in the same direction, the deflection of the spot of light was 350 divisions of the scale, but when their electromotive forces acted in opposite directions, the deflection was only 235 divisions. Compare their electromotive forces.

Answer. A : B :: 5 : 1, nearly.

N.B. This is sometimes known as Wiedemann's method.

(4.) A tangent galvanometer, a Wheatstone's rheostat, and a Daniell's cell, were joined up in simple circuit, and by adjusting the resistance a deflection of 45° was obtained. On introducing 30 more turns of the rheostat wire, the deflection was reduced to 40°. A Grove's cell was then substituted for the Daniell and the resistance adjusted till the deflection was 45°. To reduce the deflection to 40° it was necessary to introduce 50 turns of the rheostat wire. Compare the electromotive forces of the Daniell's and Grove's cells.

This is known as Wheatstone's method.

Let R = resistance of circuit in first case.
r = „ „ one turn of the rheostat wire.
E = electromotive force of a Daniell's cell.

Then by Ohm's law

$$\left.\begin{aligned}\frac{E}{R} &= m \tan 45^\circ\\ \frac{E}{R+30r} &= m \tan 40^\circ\end{aligned}\right\}; \therefore \frac{E}{\frac{E}{m}+30r} = m \tan 40^\circ \; . \; .. \; (1)$$

Similarly, for the Grove's cell we shall have

$$\frac{E'}{\frac{E'}{m}+50r} = m \tan 40^\circ \; . \; . \; . \; . \; . \; . \; (2)$$

From (1) and (2) we get

$$\frac{E'}{E'+50mr} = \frac{E}{E+30mr}; \text{ whence } \frac{E'}{E} = \frac{5}{3}.$$

(5.) Two batteries, A and B, were joined up successively with a resistance box and a tangent galvanometer, and in each case the resistance of the circuit was adjusted till the deflection was 20°. To reduce the deflection to 10° the resistance in the box of coils had to be increased by 3 ohms when the battery A was in circuit, and by 5 ohms when B was in circuit. Compare the electromotive forces of the batteries A and B. *Answer.* A : B :: 3 : 5.

(6.) A battery, A, was joined up with a Thomson's galvanometer and a resistance of 2530 ohms, when a deflection of 230 scale divisions was obtained. On increasing the resistance of the circuit by 450 ohms the deflection fell to 200. When another battery, B, was substituted for A, a resistance of 3220 ohms had to be included from the box, so as to give a deflection of 230, while to reduce this deflection to 200 an additional resistance of 360 ohms was necessary. Compare the electromotive forces of these two batteries.

Answer. B : A :: 4 : 5.

(7.) A plate of amalgamated zinc, and another of copper, were formed into a galvanic cell, the extrapolar circuit consisting of a Wheatstone's rheostat and a galvanometer. When a solution of sulphate of copper was poured into the cell, 30 turns of the rheostat wire were required to bring the deflection of the needle from 45° to 40°; but when dilute sulphuric acid was substituted for the sulphate of copper, only 20 turns were required for the same purpose. Compare the electromotive forces in the two cases.

Answer. E : E′ :: 3 : 2.

(8.) A rheostat, a galvanometer, and a water voltameter were joined up in simple circuit with 4 equal and similar galvanic cells, and it was observed that 50 turns of the rheostat wire were required to bring the deflection of the needle down from 45° to 40°. The voltameter was then removed, and it was found that 120 turns were now required to bring the needle down from 45° to 40°. Find the amount of contrary electromotive force introduced by the voltameter. *Answer.* Let E = electromotive force of one cell.

Let e = electromotive force of voltameter.

Then $e = \frac{7\,E}{5}$

(9.) A battery, A, whose internal resistance was 65 ohms, was joined up in simple circuit with a tangent galvanometer of 100 ohms resistance and a box of resistance coils, and when the resistance from the box was 1232 ohms the deflection of the needle was 33°. Another battery, B, was then substituted for A, and in order to produce the same deflection of the needle an additional resistance of 55 ohms had to be unplugged from the box. If the resistance of B was 60 ohms, compare the electromotive forces of A and B.

This is known as the method of 'Equal Deflections.'

By Ohm's law we have

$$\left.\begin{aligned}\frac{E}{B+G+R} &= m \tan 33^\circ\\ \frac{E'}{B'+G+R'} &= m \tan 33^\circ\end{aligned}\right\} \therefore \frac{E'}{E} = \frac{60+100+1287}{65+100+1232} = \frac{1447}{1397};$$

$$\therefore E' : E :: 104 : 100, \text{ nearly.}$$

(10.) A battery, A, whose resistance was known to be 33 ohms, was connected up in simple circuit with a galvanometer of 97 ohms resistance and a box resistance of 1600 ohms, and the deflection was noted. On substituting a battery, B, whose resistance was known to be 43 ohms, an additional resistance of 237 ohms had to be included in the circuit, so as to reproduce the original deflection. Compare the electromotive forces of these two batteries.

Answer. A : B :: 114 : 100, nearly.

(11.) A battery, A, was joined up in simple circuit with a reflecting galvanometer whose resistance was 5000 ohms, a box resistance of 15000 ohms, and a deflection of 86 scale-divisions was noted. On substituting for A another battery, B, an additional resistance of 2150 ohms had to be unplugged from the box, so as to bring the deflection to 86. Compare the electromotive forces of A and B.

Answer. B : A :: 11 : 10, nearly.

(12.) Two batteries, A and B, were joined according to Poggendorff's method, on opposite sides of an adjustable resistance C D (= R), so that both sent their currents towards C. A galvanometer was included in the branch C A D and an adjustable resistance (r) in C B D. On making $r = 150$ ohms it was necessary to make $R = 2350$ ohms, so that no current should flow through the galvanometer, and on making $r = 120$ we had to make $R = 1570$ ohms so as to restore the balance. Compare the electromotive forces of A and B.

If no current flows through C A D (see fig. 13), then it is clear that the electromotive force at C must be equal to that of the battery A. Let this be E.

Then the difference of potential between C and D is that due to E, and ∴ the current in C D (= R) is $C = \frac{E}{R}$, and this must be equal to the current flowing through the battery B.

But current through battery B $= \frac{E' - E}{B + r}$, where B = resistance of battery B.

$$\left.\begin{array}{lr} \text{Hence, in the first case we have} & \dfrac{E'-E}{E} = \dfrac{B+r}{R} \\ \text{,, \quad second ,\quad ,,} & \dfrac{E'-E}{E} = \dfrac{B+r'}{R'} \end{array}\right\} ;$$

$$\text{whence } \frac{E'}{E} = \frac{\Delta R + \Delta r}{\Delta R}.$$

$$\text{In our case} \quad \left.\begin{array}{l} \Delta R = 780 \\ \Delta r = 30 \end{array}\right\} ; \therefore \frac{E'}{E} = \frac{810}{780} = \frac{27}{26}.$$

(13.) Two other batteries, A and B, were joined up according to Poggendorff's method, as described in the last Example, and it was found that with $r = 104$ ohms R had to be 110 ohms, and when r was 210, R had to be 220 ohms, so that no current should flow through the galvanometer. Compare the electromotive forces of these batteries.

Answer. B : A :: 108 : 55.

(14.) A battery, A, was joined up in circuit with an adjustable resistance R and a galvanometer, and another battery, B, was joined up to the same resistance R, so that both batteries sent a current through R in the same direction. The resistance of the battery A was 20 ohms, and when R was made equal to 680 ohms, there was no current flowing through the galvanometer. Compare the electromotive forces of the two batteries.

In this case we have $r = 0$, $B = 20$, $R = 680$;

$$\text{and } \therefore \frac{E'-E}{E} = \frac{B}{R}; \; \therefore \frac{E'}{E} = \frac{B+R}{R} = \frac{20+680}{680} = \frac{35}{34}.$$

$$\therefore A : B :: 35 : 34.$$

(15.) The terminals of a Grove's cell whose internal resistance was ·74 ohm, were connected by a uniform wire 85·51 metres long, whose resistance was 10·06 ohms. When the terminals of a Daniell's cell were connected to two points 51 metres apart on this wire, so that each cell tended to produce a current in the same direction through that part of the wire which was common to both their circuits, it was found that no current was flowing through the Daniell's cell. Find the relative electromotive forces of the two cells.

Answer. G : D :: 9 : 5.

(16.) A Thomson's galvanometer of 5000 ohms resistance had a shunt across its terminals of 1120 ohms. The terminals of a battery, A, were connected for a moment with a condenser, and this condenser was then discharged through the shunted galvanometer, and the *throw* of the needle was noted. The same condenser was then charged by another battery, B, and it was found that to get the same deflection of the spot of light on the scale, the galvanometer shunt had to be increased to 2570 ohms. Compare the electromotive forces of the two batteries.

When a battery is directly connected to a condenser, the charge which it can give to it varies directly with the difference of potential of its poles.

Let E and E' represent the electromotive forces of the two batteries, and C the capacity of the condenser, then the charges are CE and CE' respectively.

Let G = resistance of galvanometer,
S_1 = „ 1st shunt to galvanometer,
S_2 = „ 2nd „ „

then, since the discharge deflections are equal, we must have

$$\frac{C E S_1}{G + S_1} = \frac{C E' S_2}{G + S_2};$$

whence

$$\frac{E}{E'} = \frac{S_2}{S_1} \times \frac{G + S_1}{G + S_2} = \frac{2570}{1120} \times \frac{5000 + 1120}{5000 + 2570}$$

$$= \frac{2570 \times 6120}{1120 \times 7570} = 1{\cdot}855, \text{ nearly.}$$

$$\therefore E : E' :: 186 : 100, \text{ nearly.}$$

(17.) A condenser was charged by one cell of a certain battery and then discharged through a galvanometer whose resistance was 4500 ohms, and which had a shunt of 1100 ohms resistance, and the deflection was noted. The same condenser was then charged by a battery of 50 similar cells arranged in series, and in order to get the same discharge deflection, the galvanometer shunt had to be reduced to 17·7 ohms. Compare the electromotive force of the battery of 50 cells with that of this single cell.

Answer. As 50·136 : 1.

(18.) With a galvanometer of 7000 ohms resistance, and a shunt of 7000 ohms, the discharge from a condenser charged by one cell of a certain battery gave a deflection of 450 scale-divisions. The discharge from the same condenser, charged by 200 cells of the same kind, through this galvanometer with a shunt of 17 ohms, gave a *throw* of 445 scale-divisions. Find the ratio of the electromotive force of the 200 cells to that of the single cell.

Answer. As 204 : 1.

(19.) A water voltameter was connected up with a battery of 10 Grove's cells in series, and the quantity of explosive gas liberated in 5 minutes was found to be 65·5 cubic centimetres. An additional resistance of 16 ohms was then added to the circuit, and the quantity of gas liberated in 10 minutes was found to be 59 cubic centimetres. Assuming that the *back* electromotive force in the voltameter was 1·5 volt, find the electromotive force of one of these Grove's cells.

Let E = electromotive force of battery.
e = back electromotive force of voltameter.
R = resistance of circuit initially.

Then, since one ampère liberates ·1764 c.c. of explosive gas per second, we have

$$\left.\begin{aligned}\frac{E-e}{R} &= \frac{65{\cdot}5}{5\times 60\times {\cdot}1764}\text{ ampère}\\ \frac{E-e}{R+16} &= \frac{59}{10\times 60\times {\cdot}1764}\quad \text{''}\end{aligned}\right\}$$

whence

$$E-e = 16{\cdot}228,\ \therefore E = 16{\cdot}228 + 1{\cdot}5 = 17{\cdot}728\,;$$

∴ electromotive force of a Grove's cell according to this experiment

= 1·77 volt, nearly.

(20.) A battery of 20 similar cells was joined up in simple circuit with a water voltameter, and it was found that 9·2 cubic centimetres of hydrogen were evolved per minute. When an additional 15 ohms of resistance was included in the circuit, the volume of hydrogen evolved per minute was 6·2 c.c. Taking the back electromotive force as equal to 1·5 volt, find the electromotive force of one cell of the battery. *Answer.* 1·94 volt.

(21.) The current from a battery of 5 Daniell's cells arranged in series was sent through a water voltameter, and in five minutes 6 cubic centimetres of explosive gas were evolved. The current from a battery of 20 Daniell's cells, just like the first but arranged in two files of 10 each, was then sent through the same voltameter when 14·5 c.c. of explosive gas were evolved in five minutes. Compare the value of the contrary electromotive force developed in the voltameter with that of a Daniell's cell. *Answer.* 1·47 D.

(22.) Two insulated metal balls of one centimetre radius were placed at a distance of 20 centimetres from centre to centre, and, when connected by very fine wires, each with one pole of a Daniell's battery of 535 cells in series, were found to attract each other with a force of ·0025 dyne. Find in electromagnetic measure the value of the electromotive force of a Daniell's cell.

The two spheres will be charged to equal and opposite potentials, which we may represent by + v and − v, so that the difference of their potentials will be 2 v. Since the capacity of a sphere is equal to its radius, the charge on each sphere will be v.

$\therefore$ Mutual force $= \dfrac{v^2}{(20)^2} = \cdot 0025 = \dfrac{1}{(20)^2}$, $\therefore v = \pm 1$,

and the difference of potential in electrostatic measure = 2.

$\therefore$ Electromotive force of 1 Daniell in electrostatic units $= \dfrac{2}{535} = \cdot 00374$, nearly.

But one electrostatic unit of potential $= 3 \times 10^{10}$ electromagnetic units;

$\therefore$ E.M.F. in electromagnetic measure of 1 Daniell

$$\begin{aligned} &= 3 \times 10^{10} \times \cdot 00374 \\ &= 1 \cdot 122 \times 10^8 \\ &= 1 \cdot 122 \text{ volts.} \end{aligned}$$

(23.) The trap-door of a guard ring electrometer had an area of 1 square centimetre, and when the plates were uncharged a weight of one centigramme was required to bring the trap-door down flush with the guard ring. The plates were then connected with the terminals of a battery, and the trap-door was again flush with its guard ring when its distance from the movable plate was ·15 centimetre. Find the electromotive force of the battery both in electrostatic and in electromagnetic measure.

By the method of Example 1 in Electrometers we find that

$$\begin{aligned} E &= 2{\cdot}356 \text{ electrostatic units,} \\ &= 2{\cdot}356 \times 3 \times 10^2 = 706{\cdot}8 \text{ volts.} \end{aligned}$$

ELECTRO-MAGNETIC MEASUREMENT.

(1.) Find the solid angle subtended by a circular ring of 12 centimetres radius at a point situated on its axis, and at a distance of 7 centimetres from the plane of the ring.

If the eye be placed at any point in space, and a straight line passing through that point be made to travel round the contour of any circuit, this line will generate a conical surface whose vertical angle measures the apparent magnitude of the circuit as seen from that point. If we imagine a sphere of unit radius described about the point as centre, then the area of the portion of the surface of this sphere which is cut off by the cone will measure the *solid* angle.

Let Fig. 14 represent a plane section of the sphere and the circular ring, A being the centre of the ring and A B the radius. Since the area of any portion of a spherical surface cut off by two parallel planes is equal to the area of the corresponding portion of the circumscribing cylinder,

$$\begin{aligned} \text{the spherical surface } DFE &= 2\pi(1 - \cos\theta) \\ &= 2\pi \text{ vers } \theta = \omega. \end{aligned}$$

But in the present case $\tan\theta = \frac{12}{7}$,

$$\therefore \cos\theta = {\cdot}504, \text{ nearly.}$$

$$\therefore \omega = 2\pi(1 - {\cdot}504) = \pi \times {\cdot}992.$$

(2.) If the ring had been 10 centimetres in radius, and the point O 10 centimetres from it, what would have been the value of the solid angle subtended at O by the ring?

Answer. $\pi \times \cdot 586$.

(3.) What is the solid angle subtended by the sky at any point on the sea?

$$\text{In this case } \theta = \frac{\pi}{2}, \therefore \omega = 2\pi.$$

N.B.—The student should notice that the solid angle subtended by any plane contour at any point very close to its plane is 2π, and that the solid angle subtended by any closed surface at any point inside of it is numerically equal to 4π.

(4.) If a current of 4 ampères were flowing round the ring described in Example 1, what would be its potential upon a unit magnetic pole placed at the given point?

The C.G.S. electromagnetic unit of current is a current of such strength that if sent round a circle of one centimetre radius the current along an arc of one centimetre would act with a force of one dyne upon a unit magnetic pole placed at its centre.

The ampère $= \frac{1}{10}$th of a C.G.S. electromagnetic unit of current.

The potential *at* a point due to a current flowing in a closed circuit is numerically equal to the product of the current strength, the strength of the pole, and the solid angle subtended by the circuit at the pole.

$$\text{In the present case } v = -c \times \omega \times 1 = -\cdot 4 \times \pi \times \cdot 992$$
$$= -1\cdot 247, \text{ nearly.}$$

The negative sign is taken because the motion of the magnet pole is always in such a direction as to tend to reduce the potential energy between it and the circuit to a minimum.

(5.) A current of 2 ampères circulates round the wire described in Example 2. Find the potential at O due to this current. *Answer.* — ·368.

(6.) What is the strength of the magnetic field at the given point in Example 4?

Let y = radius of circle, x = distance of the point from the plane of the circle;
then, neglecting the sign, we have

$$\mathrm{V} = \mathrm{C}\omega = 2\pi\mathrm{C}\{1 - \cos\theta\} = 2\pi\mathrm{C}\left\{1 - \frac{x}{\sqrt{y^2 + x^2}}\right\}.$$

Now the strength of a magnetic field in any direction is equal to the *rate of change* of the potential in that direction. By a simple application of the Differential Calculus we have

$$\frac{d\mathrm{V}}{dx} = \frac{2\pi\mathrm{C}y^2}{(y^2 + x^2)^{\frac{3}{2}}}.$$

In the present case

$$\mathrm{C} = \cdot 4,\ y = 12,\ x = 7;$$

$$\therefore \text{ magnetic force at given point} = \frac{2\pi \times \cdot 4 \times 144}{(193)^{\frac{3}{2}}}$$
$$= \cdot 135 \text{ dyne, nearly.}$$

(7.) The diameter of a circular wire coil is 18 centimetres, and the wire makes 10 turns. Find the magnetic force at the centre of the ring, and also at a point on the axis 10 centimetres distant from the plane of the ring, when a current of 2 ampères is flowing through it.

For the magnetic force at the centre we must put $x = o$ in Example 6, and we get

$$I = \frac{2\pi\mathrm{C} \times n}{y} = \frac{2\pi \times \cdot 2 \times 10}{9} = 1 \cdot 396 \text{ dyne.}$$

In the second case we have $y = 9$, $x = 10$, and then

$$I' = \frac{2\pi n\mathrm{C}y^2}{(y^2 \times x^2)^{\frac{3}{2}}} = \frac{2\pi \times 10 \times \cdot 2 \times 81}{(81 + 100)^{\frac{3}{2}}} = \frac{324 \times \pi}{(181)^{\frac{3}{2}}}$$
$$= \cdot 418 \text{ dyne.}$$

(8.) In a certain tangent galvanometer the radius of the coil was 21 centimetres, and the wire made 21 turns. What was the intensity of the electromagnetic force at the centre of the ring, and at a point on the axis 5 centimetres distant from the plane of the coil, when a current of 3 ampères was circulating through it?

Answer. 1·885 dyne and 1·735 dyne, nearly.

(9.) A current, which was found by means of a voltameter to deliver 12·5 cubic centimetres of explosive gas per minute, was sent through a circular coil of 15 centimetres radius and containing 25 turns of wire. With what force would this current act upon a unit magnetic pole placed at a distance of 25 centimetres from the centre, on a line through it perpendicular to the plane of the coil?

Answer. ·17 dyne, nearly.

(10.) Supposing the unit magnetic pole were placed at the centre of the coil, what would be the force acting on it?

Answer. 1·25 dyne, nearly.

(11.) A small floating battery has attached to its poles a vertical coil of 50 turns of wire and 2 centimetres radius. One pole of a very long bar magnet, whose strength is 20, is placed at a distance of 10 centimetres from the centre of the coil. Find the force with which the coil is drawn towards the magnet when a current of one ampère is circulating in the wire. *Answer.* 2·37 dynes, nearly.

(12.) A ring of gutta-percha-covered wire of 10 centimetres diameter is placed vertically so that its horizontal diameter is in the surface of a tub of water, and a small magnet, of strength 20 and length 4 centimetres, is floated on a cork and placed with its axis perpendicular to the plane of the ring, and its nearer pole at a distance of 3 centimetres from it. If a current of 2 ampères be made to flow round the ring, find the force with which the magnet begins to be sucked into the ring.

Let θ and θ' represent the angles which the vertical radius of the ring subtends at the two poles of the little magnet; then, since $\sin\theta = \dfrac{y}{(x^2+y^2)^{\frac{1}{2}}}$, we find by the method adopted in the solution of Example 6, that the resultant attractive force on the little magnet is

$$= \frac{2\pi C m}{y}(\sin^3\theta - \sin^3\theta'),$$

$$\text{where } \sin\theta = \frac{y}{(x^2+y^2)^{\frac{1}{2}}}\,;\ \sin\theta' = \frac{y}{\{y^2+(x+l)^2\}^{\frac{1}{2}}}.$$

$$\text{But } y=5,\ x=3,\ l=4,\ \therefore \sin^3\theta = \frac{5^3}{(34)^{\frac{3}{2}}} = \cdot 6305$$

$$\sin^3\theta' = \frac{5^3}{74^{\frac{3}{2}}} = \cdot 1964.$$

$$\therefore \text{resultant force} = \frac{2\pi \times \cdot 2 \times 20}{5}(\cdot 6305 - \cdot 1964)$$

$$= \frac{2\pi \times 4}{5} \times \cdot 4341 = 2\cdot 182 \text{ dynes.}$$

N.B. The resultant force is evidently a minimum when $\theta = \theta'$, so that the little magnet will come to rest with its centre at the centre of the coil. If the current in the coil be now reversed, the little magnet will be driven out, and will then turn round and be sucked in again to the centre.

(13.) A small magnet, one centimetre long, was hung at the centre of a circular coil whose diameter was 72 centimetres, the wire making 5 turns. The plane of the coil was placed in the magnetic meridian, and on sending a current from a battery through it, the needle was deflected through an angle of 35°. Taking the numerical value of the horizontal component of the earth's magnetic force as ·175, find the strength of the current circulating in the coil.

Let I = intensity of the magnetic field produced by the current at the centre of the ring.

Let H = intensity of the horizontal field of earth's magnetic force.

The lines of force due to the current will be at right angles to the plane of the ring.

The lines of force due to the earth's horizontal force are parallel to the plane of the ring.

The intensity of the resultant magnetic field will be $R = \sqrt{I^2 + H^2}$, and its lines of force will be horizontal, and will make with the plane of the meridian an angle whose tangent is $\frac{I}{H}$. If θ be this angle, we shall have

$$\tan\theta = \frac{I}{H} = \frac{2\pi n C}{r \times H} = \frac{2\pi \times 5 \times C}{36 \times \cdot 175};$$

$$\therefore C = \frac{36 \times \cdot 175 \times \tan 35^\circ}{2\pi \times 5} = \cdot 1404 \text{ C.G.S. unit of current}$$
$$= 1\cdot 404 \text{ ampère.}$$

Since the axis of the needle will ultimately coincide with the direction of the lines of force in the resultant magnetic field, θ is evidently the angle which the needle makes with the meridian.

(14.) If with the same coil a current of 1·489 ampères deflected the needle through an angle of 36° 36′, what was the value of the horizontal intensity of the earth's magnetic force at the place of observation?

Answer. ·175, nearly.

(15.) A piece of thick wire was bent into the form of a circle of 10 centimetres radius, and exactly at the centre was suspended a very small magnetic needle. The coil was placed in the magnetic meridian, and on sending a current through it the deflection of the needle was 46°. The horizontal intensity being ·18, what was the strength of the current? *Answer.* 2·97 ampères, nearly.

(16.) A tangent galvanometer was constructed with a coil of one decimetre radius, and having 31 turns of wire. The horizontal intensity of the earth's magnetic force being

·182, find the strength of the current in ampères which produces an angular deflection of 17° in the needle.

Answer. 28·6 milliampères.

(17.) The radius of a single-ring tangent galvanometer was 40 centimetres, and the deflection produced by a certain current at a place where the horizontal intensity of the earth's magnetic force was ·18 was 10°. Find the strength of the current. *Answer.* 2·02 ampères, nearly.

(18.) Three tangent galvanometers were constructed of the same radius, namely 23 centimetres, the first having two, the second four, and the third six coils of wire, and a very small needle was suspended at the centre of each. Supposing that a current of 1·34 ampère is sent through each of them, and that the horizontal intensity of the earth's magnetic force is ·176, what would be the deflection of the needle in each case?

Answer. 22° 35′ 11″; 39° 45′ 33″; 51° 17′ 39″.

(19.) The mean radius of a tangent galvanometer was 25 centimetres, and when a current of 1·5 ampère was sent through it the needle was deflected $56\frac{1}{2}°$. Assuming that the horizontal intensity of the earth's magnetic force was ·175, find the number of turns of wire in the coil.

By the formula in Example 13 $n = 7$ turns.

(20.) A wire 20 metres long was wound into a galvanometer ring of 25 circular coils. Find the strength of current which would produce a deflection of 45° if the horizontal intensity of the earth's magnetic force be ·18.

Answer. 145·9 milliampères.

(21.) A tangent galvanometer was constructed by winding 30 metres of wire in 48 circular coils. The earth's horizontal intensity being ·18, find the strength of current which would deflect the needle 67° when the plane of the coil was in the magnetic meridian.

Answer. 139·9 milliampères.

(22.) The wire of a tangent galvanometer which made 10 turns was 880 centimetres long. What would be the value of m in the formula $C = m \tan \theta$ in order to measure currents in absolute C.G.S. electromagnetic units (1) at a place where the horizontal intensity is ·1794, and (2) at another place where it is represented by ·198?

Since $l = 2\pi r n \therefore r = \frac{l}{2\pi n}$, and therefore

$$C = \frac{r \text{H} \tan \theta}{2\pi n} = \frac{l \text{H} \tan \theta}{(2\pi n)^2} = \frac{l \text{H}}{(2\pi n)^2}, \text{ when } \theta = 45^\circ$$

$$\therefore m = \frac{l \text{H}}{(2\pi n)^2} = 3{\cdot}9989 \times 10^{-2},$$

and $$m' = 4{\cdot}4135 \times 10^{-2}.$$

(23.) If at a place where the horizontal intensity of the earth's magnetic force is ·179 a current of a certain intensity produces in a tangent galvanometer the deflection 52°, what will be the deflections produced by a current of the same intensity through the same galvanometer at two other places where the horizontal intensities are ·194 and ·201?

Answer. 49° 45′ and 48° 44′.

(24.) The ring of a tangent galvanometer was placed approximately in the magnetic meridian, and on sending a current through it in one direction a deflection of 33° to the right was produced, and when the current was reversed the deflection was 31° to the left. If the radius of the ring was 16 centimetres and the earth's horizontal intensity ·18, what was the current strength?

Let OB (see Fig. 15) be the position of the coil, ON the magnetic meridian, OA the position of the magnet.

Let $\text{NOB} = \phi$, $\text{BOA} = \theta_1$, $\text{BOA}', = \theta_2$.

Then if I be the intensity of the magnetic field due to the current, we shall have on taking moments about 0,

$$I \cos \theta_1 = \text{H} \sin (\theta_1 + \phi).$$

Similarly, for the other deflection θ_2 we shall have

$$I \cos \theta_2 = \text{H} \sin (\theta_2 - \phi),$$

whence $\tan\phi = \frac{1}{2}(\tan\theta_2 - \tan\theta_1) = \frac{1}{2}(\cdot 6494 - \cdot 6009) = \cdot 0242$
$= \tan 1^\circ\ 23'$, nearly.

$$\therefore\ i = \text{H}\,\frac{\sin 32^\circ\ 23'}{\cos 31^\circ}$$

$$\therefore\ \text{C} = \frac{r\text{H}}{2\pi}\,\frac{\sin 32^\circ\ 23'}{\cos 31^\circ} = \frac{16 \times \cdot 18 \times \sin 32^\circ 23'}{2\pi \times \cos 31^\circ}$$

$$= \cdot 02864 \text{ C.G.S. unit}$$
$$= \cdot 286 \text{ ampère.}$$

(25.) On sending a current of a certain strength through a tangent galvanometer a deflection of 27° to the *right* was produced, and on reversing the current a deflection of 28° 30′ to the *left*. Through what angle should the plane of the coil have been turned so as to make it coincide exactly with the magnetic meridian? *Answer.* 58′, nearly.

(26.) A tangent galvanometer, constructed on Gaugain's model, consisted of 5 circular coils of wire of 16 centimetres radius. A small magnetic needle was freely suspended so that its centre was over the centre of a graduated disc on the axis of the ring, and at a distance from it of half the radius. Assuming that the earth's horizontal intensity was ·18 and that the plane of the coil was parallel to the magnetic meridian, find the angular deflection of the needle when a current of one ampère was circulating.

The object of Gaugain's form of galvanometer is to secure a uniform field in the neighbourhood of the magnetic needle, and since the needle is very small we may assume, without appreciable error, that the strength of the magnetic field due to the current in the coil is uniform throughout its length. If m represents the magnetic moment of the needle, the moment of the couple tending to twist the little magnet away from the meridian is

$$\frac{2\pi n \text{C} y^2 m}{(x^2 + y^2)^{\frac{3}{2}}},$$

and if the little magnet be deflected through an angle θ from the plane of the magnetic meridian under the joint influence of the current and of the earth's magnetic force, then

$$\text{H}\, m \sin\theta = \frac{2\pi n \text{C} y^2 m}{(x^2+y^2)^{\frac{3}{2}}} \times \cos\theta\,;$$

$$\text{or} \qquad \tan\theta = \frac{2\pi n y^2}{(x^2+y^2)^{\frac{3}{2}}} \times \frac{\text{C}}{\text{H}}.$$

In the present case

$$x = 8,\ y = 16,\ n = 5,\ \text{C} = {\cdot}1,\ \text{H} = {\cdot}18.$$

$$\therefore \tan\theta = \frac{2\pi \times 5 \times 16^2}{(256+64)^{\frac{3}{2}}} \times \frac{{\cdot}1}{{\cdot}18} = {\cdot}7805.$$

$$\therefore \theta = 37^\circ\, 58', \text{ nearly.}$$

(27.) In another Gaugain's tangent galvanometer the radius of the coil was 12 centimetres, the wire made 2 turns and the centre of the needle was on the axis of the coil, and at the distance from it of 6 centimetres. The horizontal intensity being ·18, find the strength of the current which produces a deflection of 30°.

Answer. 1·387 ampère, nearly.

(28.) In a Helmholtz's galvanometer the needle was placed at the centre between two equal circular coils of 16 centimetres radius, which was also the distance between the coils. The earth's horizontal intensity being ·18, find the strength of the current which would produce a deflection of 25°.

The intensity of the magnetic field at the centre, due to the two coils, is

$$\frac{4\pi \text{C} y^2}{(x^2+y^2)^{\frac{3}{2}}} = \frac{32\pi \text{C}}{y\sqrt{125}},$$

and when equilibrium is obtained we shall have, as in Example 26,

$$\begin{aligned} \text{C} &= \frac{y\sqrt{125}}{32\pi} \times \text{H} \tan\theta \\ &= \frac{16 \times \sqrt{125}}{32\pi} \times {\cdot}18 \times \tan 25^\circ = {\cdot}1494 \text{ C.G.S. unit} \\ &= 1{\cdot}5 \text{ ampère, nearly.} \end{aligned}$$

N.B. The object of Helmholtz's arrangement of the galvanometer is to secure a magnetic field of great uniformity where the needle is situated.

(29.) Find the value of the galvanometer *constant* in the last Example.

The galvanometer *constant* is numerically equal to the intensity of the magnetic field produced at the needle, when one C.G.S. unit of current is circulating in the coils. For Helmholtz's galvanometer, where $x = \frac{y}{2}$, we have

$$\text{constant} = \frac{4\pi y^2}{(x^2 + y^2)^{\frac{3}{2}}} = \frac{32\pi}{y\sqrt{125}} = \frac{2\pi}{\sqrt{125}} = \cdot 5619.$$

(30.) The coil of a tangent galvanometer was placed at right angles to the magnetic meridian, and the number of oscillations was counted which the needle made in 20 seconds, under the influence of the earth's magnetism alone. When a current of strength C was sent through the galvanometer the same number of oscillations was executed by the needle in 15 seconds, and with a current C′ the same number was made in 10 seconds. Compare the strengths of the currents C and C′.

In the same way as in a similar Example in the chapter on Magnetism we find that

$$\frac{C'}{C} = \frac{\frac{1}{10^2} - \frac{1}{20^2}}{\frac{1}{15^2} - \frac{1}{20^2}} = \frac{10 \times 30}{5 \times 35} \times \left(\frac{15}{10}\right)^2 = \frac{27}{7}.$$

(31.) In another experiment with the galvanometer in the same position, the needle performed a certain number of oscillations in 20 seconds when no current was passing. The same number was performed in 10 seconds when a current A was circulating in the coil, and in 5 seconds when a current B was sent round the coil. Compare the strengths of the currents A and B.

Answer. A : B :: 1 : 5.

(32.) A tangent galvanometer, consisting of a single ring whose diameter was 38 centimetres, was placed in the magnetic meridian and joined up with a battery and a voltameter, and it was found that a current which liberated 70 cubic centimetres of explosive gas per minute produced a deflection of 45°. What was the value of the horizontal intensity of the earth's magnetic force at the time and place of observation? *Answer.* ·22, nearly.

(33.) A tangent galvanometer, whose radius was 15 centimetres and which was composed of 8 turns of wire, was placed with its plane in the magnetic meridian and was joined up with a resistance box and a voltaic cell, whose electromotive force was 1·457 volts. When the resistance of the circuit, determined independently, was 4·1908 ohms, the deflection of the needle was 33°. Find the value of the horizontal component of the earth's magnetic force.

Answer. ·179.

(34.) In the year 1865 an experiment similar to the one described in the last Example was made with a galvanometer of 15 turns of wire, having a mean diameter of 20 centimetres. The cell used was a Daniell, whose electromotive force was 1·12 volt. The resistance of the circuit was 11·75 ohms and the observed deflection 27°. What was the value of the horizontal component of the earth's magnetic force at the time and place of observation?

Answer. ·176.

(35.) Two coils of 5 centimetres radius and containing 15 turns of wire were hung in parallel planes, so that the distance between their centres was 4 centimetres. A current which, in a voltameter, could evolve 23 cubic centimetres of explosive gas per minute was sent through both coils in opposite directions. Find the force in dynes with which they would repel each other.

$$\text{The strength of current in each coil} = \frac{23}{60 \times \cdot 1764}$$

$$= 2 \cdot 173 \text{ ampères.}$$

Referring to Maxwell's 'Electricity and Magnctism,' Art. 699, we find that for two parallel circles on the same axis the general expression for the attraction between them when unit current is flowing in each is

$$\frac{d\mathrm{M}}{db} = \pi^2 \frac{\mathrm{A}^2 a^2}{\mathrm{C}^4} \left\{ 2 \cdot 3 \frac{\mathrm{B}}{\mathrm{C}} + 2 \cdot 3 \cdot 4 \frac{\mathrm{B}^2 - \frac{1}{4}\mathrm{A}^2}{\mathrm{C}^3} b + \text{\&c.} \right\};$$

where $a =$ radius of small circle; $b =$ distance of its plane from the origin of co-ordinates; and $c = \sqrt{a^2 + b^2}$.

Also A, B, C are the corresponding quantities for the larger circle.

If we take the origin of co-ordinates at the centre of the smaller circle we get $b = 0$, and the above reduces to

$$\frac{d\mathrm{M}}{db} = \frac{6\,\pi^2 \mathrm{A}^2 a^2 \mathrm{B}}{\mathrm{C}^5}.$$

But in our case $\mathrm{A} = a = 5$; $\mathrm{B} = 4$; $\mathrm{C} = \sqrt{5^2 + 4^2}$;

$$\therefore \frac{d\mathrm{M}}{db} = \frac{6\,\pi^2 \times 4 \times 5^4}{(41)^{\frac{5}{2}}};$$

and since there are 15 turns of wire in each coil, and a current of ·2173 C.G.S. unit, the force between the two coils in dynes is

$$= \frac{15^2 \times (\cdot 2173)^2 \times 6\,\pi^2 \times 4 \times 5^4}{(41)^{\frac{5}{2}}} = 146 \text{ dynes, nearly.}$$

(36.) A circular conductor of 4 centimetres mean radius, and containing 9 turns of wire, was suspended horizontally from one arm of a delicate balance and counterpoised. Below and parallel to this coil, so that their centres were in the same vertical line, and at a distance of one centimetre, was a fixed coil of the same mean radius as the first, but having 11 turns of wire. A current which could liberate 17 cubic centimetres of explosive gas per minute was then sent through both coils in opposite directions. Find the additional weight in grammes (taking $g = 981 \cdot 17$) which would have to be placed on the suspended coil so as to maintain it in its original position.

As in the last Example, we have

$$\text{F} = \frac{(\cdot 1606)^2 \times 11 \times 9 \times 6\pi^2 \times 4^4 \times 1}{17^{\frac{5}{2}} \times 981 \cdot 17} = \cdot 033 \text{ gramme}$$

$$= 33 \text{ milligrammes.}$$

(37.) In another experiment with the same coils the further extremity of the balance beam was secured by a thread, and a weight of one centigramme was placed upon the upper coil. Find the strength of the current which, on being sent through both coils in opposite directions, would permit of the string being cut without its affecting the equilibrium of the coil.

Answer. 883 milliampères, nearly.

(38.) Two small wire circles, the lengths of whose radii were 2 and 3 centimetres respectively, were placed at a distance of one metre apart, their planes being at right angles to each other. When a current of 10 ampères was sent through both circuits, calculate the moment of the couple acting between them and tending to set their planes parallel.

Since the action at a considerable distance of a small circular current is the same as that of a small magnet having its axis perpendicular to the plane of the current, and whose moment is equal to the product of the current strength into the area of the circuit, we shall have

$$\text{M} = \frac{\pi r^2 \text{C} \times \pi r'^2 \text{C}}{d^3} = \frac{\pi^2 \text{C}^2 (r r')^2}{d^3} = \frac{\pi^2 \times 1^2 \times 36}{(100)^3}$$

$$= 3 \cdot 55 \times 10^{-4}.$$

(39.) If, instead of a single ring of wire, each of the above circles was composed of 100 turns of wire, find the moment of the couple tending to set their planes parallel when a current of one C.G.S. unit was sent round each.

Answer. 3·55.

(40.) What would be the magnetic moment of each of two small magnets which at the distance of one metre would exert the same action on one another as the two circuits mentioned in Example 38?

Answer. 12·566 and 28·274, nearly.

(41.) An isolated wire was wound round a wooden cylinder whose length was 29·5 centimetres. The circumference of the cylinder was 12·9 centimetres and the wire made 300 turns. If a current of unit strength were sent round this solenoid, what would be the force which it would exert on a unit magnetic pole placed on the axis at a distance of 2 centimetres from the nearer end?

In Fig. 16 let $\mathrm{OB} = x$, $\mathrm{BC} = 2r$, $\mathrm{AB} = a$, $\mathrm{COB} = \psi$, $\mathrm{DOA} = \psi'$; then if n be the number of turns per unit of length and c the current strength, the strength of the magnetic field at O is $2\pi n \mathrm{c}(\cos\psi' - \cos\psi)$.

In the present case

$$\cos\psi' = \frac{x+a}{\sqrt{r^2+(x+a)^2}} = \frac{29{\cdot}5+2}{\sqrt{4{\cdot}2+992{\cdot}25}} = \frac{31{\cdot}5}{\sqrt{996{\cdot}45}}$$
$$= {\cdot}99789\,;$$

$$\cos\psi = \frac{x}{\sqrt{r^2+x^2}} = \frac{2}{\sqrt{4{\cdot}2+4}} = \frac{2}{\sqrt{8{\cdot}2}} = {\cdot}69843\,;$$

$$\text{also } n = \frac{300}{29{\cdot}5} \text{ and } \mathrm{c} = 1\,;$$

$$\therefore \text{ force at given point} = \frac{2\pi \times 300}{29{\cdot}5}({\cdot}99789 - {\cdot}69843)$$
$$= 19 \text{ dynes, nearly.}$$

(42.) Two solenoids were constructed, one of them being 12 centimetres long, 3 centimetres in diameter, and having 480 turns of wire; while the other was 15 centimetres long, 2 centimetres in diameter, and had 810 turns of wire. These solenoids were suspended with their axes in the same straight line at a distance of 2 centimetres apart, and a current of one ampère was sent through both of them in the same direction. Find the force of attraction between them.

When the distance between the solenoids is small compared with their lengths, the force of attraction between them is

$$\text{F} = \frac{\text{C} n \pi r^2 \times \text{C}' n' \pi r'^2}{2 d^2}\,[1]$$

where C = strength of current in first solenoid,
n = number of turns of wire per centimetre,
r = radius of the coil in centimetres,
d = distance between the near ends of the two coils,
and similar expressions for the other coil.

In the present case

$$\text{C} = \text{C}' = {\cdot}1$$

$$\left.\begin{aligned} n &= \frac{480}{12} = 40 \\ n' &= \frac{810}{15} = 54 \end{aligned}\right\} \quad \left.\begin{aligned} r &= 1{\cdot}5 \\ r' &= 1 \end{aligned}\right\}$$

$$\therefore \text{ force} = \frac{({\cdot}1)^2 \times \pi^2 \times 40 \times 54 \times (1{\cdot}5)^2}{2 \times 4}$$

$$= 60 \text{ dynes, nearly.}$$

(43.) A solenoid was constructed of 1000 turns of wire, the spiral being 30 centimetres long and 13·2 centimetres in circumference. It was suspended freely, and at a distance of 20 centimetres was placed a fixed circular coil 5 centimetres in radius, having 20 turns of wire, with its plane at right angles to the axis of the solenoid. A current of 10 ampères was then sent through the coil and solenoid. Find the force of attraction between them.

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[1] Wuellner's *Experimental Physics*, vol. iv. pp. 711 and 722.

By the methods employed in previous examples the

$$\text{force} = \frac{n \pi c r^2}{(r^2 + x^2)^{\frac{3}{2}}} \times n' \pi c r'^2,$$

$$\text{where } n = 20 \qquad n' = \frac{100}{3}$$

$$r = 5 \qquad r' = \frac{13{\cdot}2}{2\pi}$$

$$x = 20, x^2 + r^2 = 425$$

$$c = {\cdot}1.$$

$$\therefore \text{force} = \frac{20 \times \pi \times 1 \times 25 \times \frac{100}{3} \times \left(\frac{13{\cdot}2}{2\pi}\right)^2}{(425)^{\frac{3}{2}}}$$

$$= 82{\cdot}872 \text{ dynes, nearly.}$$

(44.) To determine by Kohlrausch's method the value of the horizontal intensity of the earth's magnetic force, a coil was hung up by means of a bifilar suspension so that its plane coincided with the magnetic meridian, and a current of ·36717 ampère was sent through it, producing a deflection of 3° 5′·78. The sum of the areas enclosed by the coils was 297743 square centimetres and the moment of inertia of the coil was found by an independent observation to be 4323310. The mean time of oscillation of the coil, when no current was passing, was found to be 33·8 seconds. Find the value of the horizontal intensity of the earth's magnetism.

$$\left.\begin{array}{l} \text{Let } K = \text{moment of inertia of the coil} \\ \quad A = \text{sum of areas of all the convolutions} \\ \quad C = \text{current strength} \\ \quad T = \text{time of a single vibration} \\ \quad \theta = \text{angle of deflection from meridian} \end{array}\right\}.$$

Then, by the method described in Maxwell's 'Electricity,' vol. ii. page 327,

$$H = \frac{\pi^2 K}{T^2 A C} \tan \theta = \frac{\pi^2 \times 4323310 \times \tan 3^\circ\ 5'{\cdot}78}{(33{\cdot}8)^2 \times 297743 \times {\cdot}036717}$$

$$= {\cdot}185, \text{ nearly.}$$

N.B.—The student should notice that in this experiment the *sum* of the areas enclosed by the coils is not $n \pi r^2$, because the coil had become bent into an ellipse by its own weight.

(45.) Suppose that on another occasion, and at another place, a deflection of 2° 16′·42 was produced when a current of ·2782 ampère was sent through the coil, find the horizontal intensity. *Answer.* ·179.

(46.) The soft iron armature of a telegraphic instrument was originally at a distance of ·5 millimetre from the electromagnet. It was then adjusted so that when no current was flowing its distance from the electromagnet was one millimetre and the tension of the opposing spring was increased fourfold. Compare the magnetic forces required in the two cases to pull down the armature.

Let f_1 and f_2 be the required magnetic forces; then, since the force required to attract the armature varies directly as the resistance of the spring and as the square of the distance, we must have

$$\frac{f_2}{f_1} = \left(\frac{\cdot 1}{\cdot 05}\right)^2 \times \frac{4}{1} = \frac{16}{1}.$$

(47.) In one of Du Moncel's experiments the *pull* of an electromagnet on its armature at a distance of one millimetre was equal to the weight of 15 grammes. Express in gravitation units the magnetic force required to attract the armature when at a distance of two millimetres, the resistance of the antagonistic spring being reduced by one half.

Answer. 30 grammes.

(48.) The electromagnet of a relay instrument is successively excited by currents of 5 and of 2 milliampères. Compare the attractive forces exerted upon the soft iron armature in the two cases.

Let f_1 and f_2 be the forces; then, if c_1 and c_2 be the corresponding currents, we have by Dub's law

$$\frac{f_2}{f_1} = \left(\frac{c_2}{c_1}\right)^2 = \frac{25}{4} = \frac{6\cdot 25}{1}.$$

(49.) The poles of a battery of 8 Daniell's cells in series were connected in multiple arc to two telegraphic instruments, the electromagnets of which had each a resistance of 22 ohms, and when their armatures were at one millimetre distance, they were attracted with a force equal to the weight of 70 grammes. Assuming that the electromotive force of each cell was 1·12 volt, and its resistance 11 ohms, find the attraction of each of these electromagnets for its armature when the other instrument is thrown out of circuit.

Let c_1 = current through each electromagnet in first case;

Let c_2 = current through each electromagnet in second case;

then $2\,c_1$ = current through battery in first case.

$$\therefore 2\,c_1 = \frac{8 \times 1{\cdot}12}{11 + 88} = \frac{8{\cdot}96}{99} \text{ ampères} = 90{\cdot}5 \text{ milliampères};$$

$$\therefore c_1 = 45{\cdot}25 \text{ milliampères}$$

$$c_2 = \frac{8 \times 1{\cdot}12}{22 + 88} = \frac{8{\cdot}96}{110} \text{ ampères} = 81{\cdot}45 \text{ milliampères};$$

$$\therefore f_2 = \left(\frac{81{\cdot}45}{45{\cdot}25}\right)^2 \times 70 = 227 \text{ grammes.}$$

(50.) If the battery had been arranged in two files of 4 cells each, and the similar terminals connected to one of the above electromagnets, find the attractive force on its armature in gravitation units. *Answer.* 354·3 grammes.

(51.) The resistance of a certain thermo-electric pile was ·4 ohm, while that of the galvanometer to be used with it was ·04 ohm. Had the coil contained the same mass of wire, but of one third the diameter, compare the magnetic effects of the current in the two cases, supposing that the electromotive force remains constant.

The strength of each pole of an electromagnet is directly proportional to the strength of the magnetising current and to the number of turns of wire in its coils. When the wire is made of one-third the original diameter, its length will be increased 9 times and its resistance will be 81 times as great as it was before. At the same time there will be 9 times as many coils in the galvanometer.

Hence, if E = electromotive force of pile and n = number of coils,

$$\left.\begin{aligned} m_1 &= \frac{n\,\mathrm{E}}{{\cdot}4 + {\cdot}04} = \frac{n\,\mathrm{E}}{{\cdot}44} \\ m_2 &= \frac{9\,n\,\mathrm{E}}{{\cdot}4 + 81 \times {\cdot}04} = \frac{9\,n\,\mathrm{E}}{3{\cdot}64} \end{aligned}\right\} \therefore \frac{m_2}{m_1} = \frac{9}{3{\cdot}64} \times \frac{{\cdot}44}{1} = \frac{109}{100}.$$

(52.) A line of telegraph A B is worked by a battery at A, the joint resistance of the line and battery being 160 ohms. Compare the intensities of the electromagnetic effects which can be produced at B (1) by using a coil of 1600 turns of wire and 160 ohms resistance, (2) by using a coil of 400 turns of wire and only 10 ohms resistance.

Answer. As 2·125 : 1.

(53.) If the line and battery had a resistance of 50 ohms, compare the magnetic effects that could be produced at B with the above two coils. *Answer.* As 8 : 7.

(54.) If the line and battery had a resistance of only 10 ohms, compare the electromagnetic effects which could be obtained at B with these two coils.

Answer. As 8 : 17.

(55.) A mass of 4000 grammes of copper is to be drawn out into wire so as to form the coil of an electromagnet. The resistance of the voltaic cell which is to be used to excite this coil is equal to 11·1 metres of wire of one millimetre diameter made of this copper. What should be the diameter of the wire so as to obtain the maximum electromagnetic effect, taking the specific gravity of copper equal to 8·88?

Since the electromagnetic effect varies directly with the strength of the magnetising current, it follows that the maximum effect will be produced when the resistance of the coil is equal to that of the cell.

Let m_1 = mass of copper wire 11·1 metres long and one millimetre in diameter;

Let m_2 = mass of copper wire x millimetres in diameter; then $m_1 = \pi \times \cdot 0025 \times 1110 \times 8{\cdot}88$ grammes;

$m_2 = 4000$;

hence, by the method given in the Examples on resistance of wires,

$$1 = \frac{R_2}{R_1} = \frac{m_2}{m_1} \times \left(\frac{1}{x}\right)^4;$$

$$\therefore x = \sqrt[4]{\frac{4000}{\pi \times \cdot 0025 \times 1110 \times 8{\cdot}88}}$$

$$= 2{\cdot}681 \text{ millimetres.}$$

(56.) The resistance of a certain voltaic cell is ·126 ohm, while that of a copper wire one metre long and one millimetre in diameter is $\frac{1}{48{\cdot}5}$ ohm. A mass of 4000 grammes of copper is to be drawn into wire to make a coil to be excited by this cell. What must be the diameter of the wire so as to obtain the greatest electromagnetic effect?

Answer. 3·112 millimetre, nearly.

(57.) A single voltaic cell was connected with a single ring tangent galvanometer whose radius was 20·4 centimetres, and the mean deflection of the galvanometer needle during the experiment was 27°. The zinc was weighed before and after the experiment, and the loss during the five minutes that the experiment lasted was ·2982 gramme. Taking the horizontal intensity of the earth's magnetic force at ·1827, calculate from this experiment the value of the electrochemical equivalent of zinc.

The mean current strength during the time the experiment lasted is given by the formula

$$C = \frac{\text{H.R.} \tan\theta}{2\pi} = \frac{\cdot 1827 \times 20{\cdot}4 \times \tan 27^\circ}{2\pi}$$

$$= \cdot 302241 \text{ C.G.S. unit,}$$

and the electrochemical equivalent of zinc being the mass of zinc dissolved in one second by the C.G.S. unit of current is

$$= \frac{\cdot 2982}{300 \times \cdot 302241} = \cdot 00329, \text{ approximately.}$$

(58.) In another experiment made at the same place the mass of zinc dissolved in five minutes was ·5055 gramme, while the mean deflection of the galvanometer needle was 40° 12′. Calculate from the data of this experiment the electrochemical equivalent of zinc.

Answer. ·00336, nearly.

(59.) In an experiment made at Manchester in 1845 by Joule a tangent galvanometer of one foot diameter was connected up with a battery and an electrolytic cell containing a solution of sulphate of zinc, and it was found that when the angular deflection of the needle was 36° 18′, the mass of zinc deposited per second was ·01508 grain. Taking the horizontal intensity of the earth's magnetic force at Manchester at that date as ·1633, calculate the value of the electrochemical equivalent of zinc.

Radius of galvanometer in centimetres

$$= \frac{30 \cdot 4797}{2} = 15 \cdot 2398\,;$$

mass of zinc deposited per second

$$= \cdot 01508 \times \cdot 064799 \text{ gramme}$$
$$= 9 \cdot 7717 \times 10^{-4} \text{ gramme.}$$

If c = strength of current in C.G.S. units; then

$$c = \frac{\mathrm{H}\,r}{2\,\pi} \tan\theta = \frac{15 \cdot 2398 \times \cdot 1633 \times \tan 36^\circ\, 18'}{2\,\pi}$$
$$= \cdot 29096 \text{ C.G.S. unit.}$$

Hence the electrochemical equivalent of zinc

$$= \frac{9 \cdot 7717 \times 10^{-4}}{\cdot 29096} = \cdot 003358.$$

(60.) In another experiment the electrolytic cell contained acidulated water, and the mass of hydrogen liberated per second was ·001092 grain, while the angular deflection of the needle of the tangent galvanometer was 60° 24′. Find from these data the value of the electrochemical equivalent of hydrogen. *Answer.* ·0001015.

MAGNETO-ELECTRIC INDUCTION.

(1.) On a broad-gauge line the width between the rails was 6 feet, and a locomotive engine was travelling along it at the rate of 40 miles an hour. Find the electromotive force between the wheels on opposite sides of the engine due to cutting the lines of terrestrial magnetic force, the vertical intensity being ·39 in C.G.S. measure.

When a complete circuit or any portion of it changes its position in a magnetic field so that there is a change in the number of lines of force enclosed by the circuit, this variation tends to produce a current whose strength, by Ohm's law, is

$$\mathrm{C} = \frac{\mathrm{E}}{\mathrm{R}};$$

where R = total resistance of the circuit, and E is the rate at which the number of lines of force which thread the circuit increases or diminishes, so that $\mathrm{E} = \frac{d\mathrm{N}}{dt}$.

In the present case the velocity of the train in centimetres per second = 40 × 44·704;

Also, 6 feet = 6 × 30·4797 centimetres.

But the vertical intensity being ·39 in C.G.S. measure, 39 lines of force pass through every 100 square centimetres of the circuit;

$$\begin{aligned}\therefore \text{E} &= 40 \times 44{\cdot}704 \times 6 \times 30{\cdot}4797 \times {\cdot}39\\ &= 127536 \text{ electromagnetic units}\\ &= 127536 \times 10^{-8} \text{ volt}\\ &= 1275{\cdot}36 \text{ microvolts.}\end{aligned}$$

(2.) Find the electromotive force between the wheels on opposite sides of a railway carriage which is travelling at the rate of 50 miles an hour on a narrow-gauge line (4 feet $8\frac{1}{2}$ inches) due to cutting the lines of terrestrial magnetic force, the vertical intensity being ·438.

Answer. 1041·7 microvolts.

(3.) Two long and thick metal rails, A B, C D (see Fig. 17) are fixed parallel to each other in a plane at right angles to the line of *dip* and at a distance of one metre apart. They are connected at the ends A and C by a galvanometer of ·5 ohm resistance, and at two other points by a movable metal cross piece. The total magnetic intensity in C.G.S. units being ·472, calculate the strength of the current which will flow through the galvanometer when the cross bar is made to move at the rate of one metre per second parallel to the rails.

$$\begin{aligned}\text{Here E} &= {\cdot}472 \times 100 \times 100\\ &= 4720 \text{ electromagnetic units}\\ &= {\cdot}472 \times 10^{-4} \text{ volt};\end{aligned}$$

$\therefore$ current through galvanometer

$$= \frac{{\cdot}472 \times 10^{-4}}{{\cdot}5 \times 10^{9}} = 9{\cdot}44 \times 10^{-14} \text{ ampère.}$$

(4.) What would have to be the velocity of the cross wire in metres per second so as to develop an electric current of one milliampère?

Answer. 106×10^{7} metres per second.

(5.) Suppose that in Example 1 the plane of the rails was inclined to the line of dip at an angle of 35°, find the electromotive force between the opposite ends of the cross rail when it was moved along the fixed rails at the rate of 10 metres a second.

The component of the motion perpendicular to the line of dip is 1000 × sin 35°, and ∴ the electromotive force developed by the motion is

$$\begin{aligned} \mathrm{E} &= \cdot 472 \times \sin 35^\circ \times 10^6 \\ &= 2\cdot 707 \times 10^5 \text{ electromagnetic units of potential,} \\ &= 2707 \text{ microvolts.} \end{aligned}$$

(6.) On a certain occasion Faraday immersed two platinum plates below the surface of the Thames, near Waterloo Bridge, at a distance of 960 feet apart, across the stream, when the current was at the rate of one mile an hour. Assuming that at the time and place of observation the vertical component of the earth's magnetic force was ·4611 C.G.S. units, what should be the value of the electromotive force developed between the two platinum plates?

If we imagine a line passing from one side of a river to the other through the water, and returning through the land beneath the water, it traces out a circuit of conducting matter, one part of which, when the water moves up or down the channel, is cutting the magnetic curves of the earth, whilst the other is relatively at rest; and if we conceive a mass of air or water, or any other substance moving relatively to the earth with a velocity v, and if A and B are fixed points on it or at its two sides, at a distance d apart in a line perpendicular to the direction of motion, then between A and B, or between any fixed conductors connected with them, and insulated in all other places from the moving mass, there will be an electromotive force whose value is

$$\mathrm{E} = I v d$$

where I is the component of the earth's magnetic force perpendicular to the plane of A B and the lines of motion across it.

In the present case,

v = one mile an hour = 44·704 centimetres per second,
d = 960 feet = 29260·5 centimetres,
I = ·4611

$$\therefore E = \cdot 4611 \times 44\cdot 704 \times 29260\cdot 5$$
= 603147 electromagnetic units of potential
= 6031 microvolts, approximately.

The student should consult Faraday's 'Bakerian Lecture,' January 1832, and Sir W. Thomson on the Mechanical Theory of Electrolysis, 'Phil. Mag.' 1851.

(7.) If the stream were running at the rate of four miles an hour, compare the electromotive force produced by the motion with that of a Daniell's cell which is assumed to be 1·12 volt. *Answer.* ·0215 of a Daniell, nearly.

(8.) Assuming that the minimum electromotive force which is requisite for effecting the electrolysis of water is 1·5 volt, find the velocity of the current which would have been necessary in order to produce between the platinum plates an electromotive force sufficient for this purpose.

Answer. 248·7 miles an hour, nearly.

(9.) In the 'Phil. Mag.,' 1851, Sir W. Thomson suggested as an experiment that the two ends of the cable wires across the Straits of Dover should be connected with platinum plates immersed in the sea on opposite sides of the Channel. Taking the distance between the plates at 20 miles in a direction generally at right angles to the direction of motion of the water in the Channel, and that in a particular state of the tide the average velocity of the current was a mile an hour, what electromotive force would be produced between the plates, the vertical intensity being ·461.

Answer. ·663 volt, nearly.

(10.) A metal disc, touched at its centre and circumference by fixed wires, was made to revolve in its own plane about a vertical axis, and the ends of the wires were separated by an insulating medium. Assuming the vertical component of the earth's magnetic force at ·46 C.G.S. unit, the radius of the disc at 1 metre and its velocity of rotation at 10 revolutions a second, calculate the electromotive force produced.

Let A and B (Fig. 18) be the ends of the fixed wires in contact with the centre and circumference of the disc; then the rate at which the lines of force are cut by the rotating radius A C is $\frac{1}{2} r^2 I \omega$;
where r = radius of disc, ω = angular velocity, and I = earth's vertical intensity;

$$\therefore E = \frac{1}{2} r^2 \omega I = \frac{(100)^2}{2} \times 2\pi \times 10 \times {\cdot}46$$

$$= 144510 \text{ electromagnetic units}$$

$$= 1445{\cdot}1 \text{ microvolts.}$$

See Faraday's 'Experimental Researches,' par. 154.

(11.) In one of Faraday's experiments a disc of copper, 3·81 centimetres in diameter, and touched at its centre and circumference by fixed wires, was made to rotate in a horizontal plane. Supposing it to have made 50 revolutions a second, what electromotive force would have been developed between the free ends of the wires?

Answer. 2·6 microvolts, nearly.

(12.) Suppose an electrical machine were constructed according to Faraday's suggestion ('Exp. Res.,' par. 158) as follows: 20 metallic discs of one foot radius are metallically connected, alternately at the edges and at the centres, by means of mercury, and are then made to revolve so that all the odd numbers go round one way and all the even numbers the other way, and that the centre of the first and the outside edge of the last disc are touched by wires whose other ends are separated by a layer of insulating material. The whole being placed so that the discs are horizontal and made to perform 35 revolutions a second, calculate the difference of potential between the free ends of the wires, the vertical component of the earth's magnetic force being ·4611 C.G.S. unit. *Answer.* 9420 microvolts, nearly.

(13.) How many discs, each of one foot radius, would be required, so that, with a rotation at the rate of 100 turns a second about a vertical axis, an electrical machine similar to the one just described might develop an electromotive force equal to that of a single Daniell's cell, assumed to be 1·074 volt? *Answer.* 799 discs.

(14.) A copper disc of one foot radius, touched at its centre and circumference by the electrodes of a single Daniel's cell, is free to turn about a vertical axis by the action of the earth's magnetism upon a current passing through it. Supposing that it were subject to no frictional or other resistance, what would be the limiting rate of its angular velocity, taking the electromotive force of a Daniell's cell at 1·12 volt and the vertical intensity of the earth's magnetic force at ·44 C.G.S. unit?

When any portion of a circuit conveying a current is allowed to obey its tendency to move in a magnetic field, then, in accordance with Maxwell's rule, it moves so as to increase the number of lines of force passing through it; and therefore, when it is subject to the earth's vertical magnetic force the result of the current flowing between the centre and circumference of the horizontal disc will be that the disc will rotate so as to increase the number of lines of force enclosed by contiguous radii. Further, assuming that there is nothing to impede the motion, the disc will revolve with a continually accelerated motion until the electromotive force developed by the motion exactly balances that of the Daniell's cell.

Let E = electromotive force required, n = number of turns per second, then

$$E = \tfrac{1}{2}(30{\cdot}48)^2 \times {\cdot}44 \times 2\pi n = 1{\cdot}12 \times 10^8, \text{ by the question;}$$

$$\therefore n = \frac{1{\cdot}12 \times 10^8}{\pi \times {\cdot}44 \times (30{\cdot}48)^2} = 87214 \text{ turns per second.}$$

(15.) If the Daniell were replaced by a thermo-electric couple whose electromotive force is 137 microvolts, find the limiting angular velocity, assuming that there is no friction.

Answer. 10·7 turns, nearly.

(16.) Suppose that in Example 3 there is placed between A and D a battery which can send a current of 10 ampères in the direction A B C D. Find the force with which the crosspiece is repelled in the direction A B.

Suppose that in a very small interval of time the slider B C has moved into the position B′ C′, so that B B′ $= x$, and if B C $= d$, the number of lines of force added to the circuit will be $I\,d\,x$, and the *work done* $=$ C $I\,d\,x$; and therefore the force acting on B C, and to which this work is due, is C $I\,d$.

In the present case the required force

$$= 1 \times {}·472 \times 100 = 47·2 \text{ dynes.}$$

(17.) If the resistance of the circuit were 4 ohms, and the electromotive force between A and D one volt, what force would have to be applied to the slider by friction or otherwise so as to keep it at rest? *Answer.* 1·18 dyne.

(18.) A wire pendulum, 5 centimetres long, has its upper end connected with one pole of a battery, which gives a current of ·5 ampère, and the other end of the wire just dips into a mercury trough, by which it is connected with the other pole of the battery. The pendulum is placed between the poles of a powerful magnet, which produces in the space occupied by the pendulum a uniform magnetic field of strength 500. Find the moment of the force with which it is jerked out of the mercury.

The pendulum being the movable portion of the circuit, the number of lines of force which are added to the circuit when the pendulum swings through a very small angle $\delta\theta$ is $\frac{1}{2} r^2 \delta\theta \times I$. Hence the work done for a displacement $\delta\theta$ is $\frac{1}{2} r^2 \delta\theta\, I$ C; and therefore the moment of the force which causes the pendulum to be jerked out of the mercury is

$$\tfrac{1}{2} r^{2} I \mathrm{C} = \frac{25}{2} \times 500 \times \cdot 05 = 312\cdot 5 \text{ dynes.}$$

That is to say, a force of 312·5 dynes applied at a point one centimetre from the axis of rotation would keep the pendulum at rest.

(19.) A similar experiment was performed with a copper spur wheel of 5 centimetres radius, which was capable of rotation about its centre. The centre was connected with one end of a battery, and the lower end of the radius, which at any instant was vertical, just touched the surface of the mercury in a trough, by which the circuit of the battery was completed, and a current of 2 ampères flowed through the circuit. If the vertical radius was between the flat poles of a horse-shoe magnet of strength 600, find the moment of the force causing the wheel to rotate.

Answer. Moment $= \tfrac{1}{2} r^{2} I \mathrm{C} = 1500$ units.

(20.) A circular ring of wire, the diameter of which is 30 centimetres, has a resistance of one microhm. On the straight line through the centre of the ring, perpendicular to its plane, and at a distance of 10 centimetres, is placed a north magnetic pole of strength 200. If in the space of a second the magnetic pole be removed to a distance of 100 centimetres from the ring, find the average strength of current developed in the ring and its direction.

When the number of lines of force which thread a conducting circuit is made to vary, the total electromotive force acting round the circuit at any instant is measured by the rate of decrease of the number of lines of force which thread it. The total electromotive force acting round the circuit during the given time is (Fig. 19)

$$2\pi m(\cos\theta' - \cos\theta),$$

since the number of lines of force passing through the circuit in the initial position is $2\pi m(1 - \cos\theta)$.

$$\left.\begin{aligned}\text{But } \cos\theta &= \frac{x}{\sqrt{a^2+x^2}} = \frac{10}{\sqrt{15^2+10^2}} = {\cdot}5547\\ \cos\theta' &= \frac{100}{\sqrt{15^2+100^2}} = \frac{100}{\sqrt{10225}} = {\cdot}9889\end{aligned}\right\};$$

$$\therefore \cos\theta' - \cos\theta = {\cdot}4342\,;$$

$$\therefore \text{average current per second} = \frac{2\pi \times 200 \times {\cdot}4342}{10^8} \div \frac{1}{10^6}$$

$$= 5{\cdot}456 \text{ ampères.}$$

With respect to the *direction* of the current, we know that the action of a circular current on a north magnetic pole placed on its axis so that the current as seen from the pole circulates in the direction of the hands of a clock is to *attract* the pole; and by Lenz's law we know that the direction of the current induced in the coil will be such that the electro-magnetic action between the current and the pole will tend to oppose their relative motion; hence, as the pole moves *away* from the coil the direction of the current in the coil as seen from the pole will be the same as that of the hands of a clock.

(21.) Suppose that in Example 20 a magnet whose length is 10 centimetres and magnetic moment 1500 was placed along the axis of the coil so that its north pole was at 10 centimetres distance from the plane of the coil and was suddenly moved towards the coil through a very small distance with a velocity of 1 metre a second. Find the strength and direction of the current induced in the coil.

If a circuit through which a current of unit strength is flowing exerts upon a magnetic pole a force whose component in any direction is f, then when no battery current is flowing in the circuit, but the magnetic pole is moved *backwards* along this direction with a velocicy v, it will induce in the circuit an electromotive force $= vf$.

Let θ and θ' be the angles which the radius of the ring subtends at the poles of the magnet, then the resultant force on the magnet is

$$f = \frac{2\,\pi\,m}{a}\,(\sin^3\theta - \sin^3\theta')\,;$$

and therefore the electromotive force developed

$$= \frac{2\,\pi\,m}{a}\,(\sin^3\theta - \sin^3\theta') \times v.$$

$$\text{But } \sin\theta = \frac{a}{(a^2 + x^2)^{\frac{1}{2}}} = \frac{15}{(15^2 + 10^2)^{\frac{1}{2}}} = \cdot832\,;$$

$$\therefore \sin^3\theta = \cdot57603\,;$$

$$\sin\theta' = \frac{a}{\{a^2 + (x+l)^2\}^{\frac{1}{2}}} = \frac{15}{(15^2 + 20^2)^{\frac{1}{2}}} = \cdot6\,;$$

$$\therefore \sin^3\theta' = \cdot216\,;$$

$$\therefore \text{E.M.F.} = \frac{2\,\pi \times 150 \times \cdot36}{15} \times 100$$

$$= 2261\cdot9 \text{ electromagnetic units}\,;$$

$$\therefore \text{current strength} = \frac{2261\cdot9}{10^8} \div \frac{1}{10^6} = 22\cdot619 \text{ ampères.}$$

(22.) If the coil contained 500 turns of wire and had a resistance of 10 ohms, and was connected with a galvanometer of 40 ohms resistance, find the strength of the momentary induced current.

Answer. ·22619 milliampère.

(23.) A Delezenne's circle is placed so that the axis of rotation is perpendicular to the line of dip at a place where the total magnetic intensity is ·47 and is rotated with a velocity of 60 turns a minute. The diameter of the ring is 60 centimetres, the wire makes 20 turns, and the resistance of the coil is 3 ohms, and that of the galvanometer with which it is connected is 20 ohms. Calculate the average strength of the current.

Let A B (Fig. 20) be the projection of the ring on the plane of the paper, and suppose the ring to revolve about an axis through O in the direction of the arrow. Then if A represent the sum of the areas of the coils and I the intensity of the field, the number of lines of force included by the coil in its initial position is $= A\,I$.

When the coil has turned through one quadrant and come into the position S N the number of lines of force which thread the circuit becomes zero, and by the time it has turned through the next quadrant the number is $-A\,I$.

Hence, during one semirevolution the total change in the number of lines of force which thread the circuit is $2\,A\,I$.

During the next semirevolution the process consists in taking out the $-A\,I$ negative lines of force and putting in $+A\,I$, so that there is again a change of $2\,A\,I$, but the induced current would be in the opposite direction in the external circuit to that which it was during the previous semirevolution. To obviate this a suitable commutator is provided which passes the current in the galvanometer always in the same direction, although its direction in the rotating coil is reversed at every semirevolution. Hence, the total current during one revolution.

$$= \frac{4\,A\,I}{R + G} = \frac{4 \times \cdot 47 \times \pi \times 30^2 \times 20}{10^8} \times \frac{1}{3 + 20}$$

$$= \frac{1 \cdot 88 \times 18 \times \pi}{23 \times 10^8} \text{ ampères}$$

$$= 1 \cdot 47 \times 10^{-2} \text{ milliampères.}$$

(24.) If the resistance of the coil were ½ an ohm and that of the galvanometer ½ an ohm, what would be the mean strength of the current when the coil made 2 revolutions a second? *Answer.* ·68 milliampère, nearly.

(25.) A copper ring 10 centimetres in mean radius and whose sectional area is one square centimetre is mounted like a Delezenne's circle and placed with its axis of rotation perpendicular to the line of dip at a place where the total magnetic intensity is ·46. If the resistance of the ring be 1015·4 microhms, and it be rotated at 1000 turns a minute, find the value of the average current induced in it?

Answer. 94·9 milliampères, nearly.

(26.) Calculate the value of the electromotive force which is induced at the instant of passing the magnetic meridian in a circular coil, of 20 centimetres diameter, having 200 turns of wire, which is revolving at 12 revolutions a second about a vertical diameter, the horizontal intensity of the earth's magnetic force being ·18, and no other magnetic influence being supposed to be present.

If at any instant the normal to the plane of the coil makes an angle θ with the magnetic meridian, then the number of lines of force which thread the coil is

$$n \pi r^2 I \cos \theta.$$

For a small angular displacement $(-\delta \theta)$ of the coil, the change in this number is $n \pi r^2 I \sin \theta \, \delta \theta$, and if this change take place in the short interval of time δt, the *rate* of change which is equal to the induced electromotive force *at* that instant is

$$n \pi r^2 I \sin \theta \frac{\delta \theta}{\delta t} = n \pi r^2 I \sin \theta \times \omega$$

where ω = angular velocity of the coil.

At the instant of passing the meridian $\theta = 90°$, and $\therefore \sin \theta = 1$, and the induced E M F

$$= n \pi r^2 I \omega = 200 \pi \times 100 \times {\cdot}18 \times 24 \pi \text{ C.G.S. units}$$
$$= 8527{\cdot}4 \text{ microvolts.}$$

(27.) A circular coil, 20 centimetres in diameter, and composed of 25 turns of insulated wire, is made to revolve about a vertical diameter at the rate of 103 revolutions in 10 seconds. Assuming that the horizontal intensity of the earth's magnetic force at the place of observation is ·179, what will be the electromotive force induced in the coil,

(1) when it coincides with, (2) when it is at right angles to, (3) when it makes an angle of 45° with, the magnetic meridian?

Answer. (1) 909·8 microvolts, (2) zero, (3) 643·5 microvolts.

(28.) A coil of wire consisting of 300 turns, the average area of each turn being 100 square centimetres, is mounted so as to rotate between the poles of an electromagnet in a nearly uniform magnetic field of strength 1000. The resistance of the coil is 3 ohms, and that of the exterior circuit, to which it is joined by a suitable commutator, is 5 ohms. Find the average strength of the current when this armature is made to revolve at 1000 turns a minute?

Answer. Average E.M.F. = 125·67 volts; ∴ average current = 15·708 ampères.

(29.) The armature of a magneto-electric machine has a resistance from brush to brush of $\frac{1}{4}$ of an ohm, and when running at 900 revolutions a minute, develops an electromotive force of 25 volts. What current would this machine send through an extrapolar resistance of 500 ohms if the armature were wound so as to have a resistance of one ohm, and were revolved at 1000 revolutions a minute?

If wound with wire of the same diameter the new armature would have 4 times the number of turns, and 4 times as great a resistance, as the old one. Hence, if it were revolved at the original speed the electromotive force would be

$$4 \times 25 = 100 \text{ volts};$$

and when revolved at the rate of 1000 revolutions a minute, the electromotive force

$$= \frac{1000}{900} \times 100 = \frac{1000}{9} \text{ volts}.$$

And the total resistance in the circuit being 501 ohms, the current

$$\doteq \frac{1000}{9 \times 501} = \cdot 22 \text{ ampère}.$$

(30.) A certain magneto-electric machine, when running at 600 revolutions a minute, produces an electromotive force of 10 volts. Calculate the value of the current which it can send through an extrapolar resistance of 30 ohms, when it is running at 800 revolutions a minute, the resistance of the revolving armature being ¾ of an ohm.

Answer. ·43 ampère, nearly.

DISTRIBUTION OF HEAT AND WORK.

(1.) A current of unit strength circulates through a coil of wire whose resistance is 1·2 ohm. What amount of heat is developed in the coil in 5 seconds?

If R = resistance of coil, E = difference of potential between its ends, and C = strength of current, then the work done between the ends of the coil in t seconds is

$$W = E\,C\,t \text{ ergs.}$$

But by Ohm's law $E = C \times R$; $\therefore W = C^2 \times R \times t$ ergs.

If J represent the mechanical equivalent of the amount of heat which is required to raise the temperature of a gramme of water from 0° C. to 1° C., commonly called *Joule's equivalent*, and if H represent the amount of heat, expressed in water-gramme degrees Centigrade (w.g.d.C.°) developed in t seconds, then we have

$$J\,H = E\,C\,t = C^2 R\,t = \frac{E^2}{R} \times t.$$

In the present case

$$R = 1{\cdot}2 \times 10^9;\ C = 1,\ J = 4{\cdot}2 \times 10^7;\ t = 5;$$

$$\therefore H = \frac{C^2 R\,t}{J} = \frac{1{\cdot}2 \times 10^9 \times 5}{4{\cdot}2 \times 10^7} = 142{\cdot}86 \text{ w.g.d. C.}$$

(2.) A current of one ampère circulates through a coil of thin wire, which has a resistance of 3 ohms. What amount of heat is developed in the wire in 5 seconds?

Answer. 3·57 w.g.d. C. = ·85 joule.

N.B. The heat which is generated in one second by a current of one ampère flowing through a resistance of one ohm is called a *joule*.

One water-gramme-degree Centigrade = $4{\cdot}2 \times 10^7$ ergs,

One joule = 10^7 ergs;

$$\therefore \text{ One joule} = \frac{\text{one w.g.d. C.}^\circ}{4{\cdot}2} = 0{\cdot}238 \text{ w.g.d. C.}^\circ.$$

(3.) Supposing that the coil had a resistance of exactly one ohm, and that a current of 2 ampères is circulating through it. What amount of heat is developed in 3 seconds?

Answer. 6 joules.

(4.) A current of 10 ampères circulates for 5 minutes through a wire whose resistance is ·9536 ohm. Find the amount of heat developed in this wire.

Answer. 28608 joules.

(5.) Find the amount of heat developed in 5 minutes by the passage of the current from a battery of 15 cells in series, through a wire of 15 ohms' resistance, each cell having an electromotive force of 1·12 volt and an internal resistance equal to 5 ohms. *Answer.* 156·8 joules.

(6.) The internal resistance of a galvanic battery is equal to the resistance of three metres of a particular wire. Compare the quantities of heat produced in a given time both inside and outside the battery, when its poles are connected by one metre of this wire, with the quantities produced in the same time when they are connected by 37 metres of the same wire.

Answer.

$$\frac{\text{Heat in battery in 1st case}}{\text{Heat in battery in 2nd case}} = \frac{100}{1}.$$

$$\frac{\text{Heat in wire in 1st case}}{\text{Heat in wire in 2nd case}} = \frac{100}{37}.$$

(7.) The extrapolar portion of a voltaic circuit consisted of a single arc formed of two different wires in series. One of them was of copper, 3 feet long and ·065 inch in diameter, while the other was of platinum, 6 inches long and ·035 inch in diameter. If the resistance of copper be to that of platinum as 112 : 1243, compare the amounts of heat developed in the two wires per second.

For a given current the heat developed in a given time varies as $\frac{\text{length} \times \text{specific resistance}}{(\text{diameter})^2}$;

$$\therefore \frac{\text{heat in platinum wire}}{\text{heat in copper wire}} = \frac{6}{36} \times \left(\frac{65}{35}\right)^2 \times \frac{1243}{112}$$

$$= \frac{6{\cdot}3796}{1}.$$

(8.) One portion of a simple circuit was composed of 10 feet of copper wire ·007 inch in diameter, and another part of 5 feet of iron wire, ·148 inch in diameter. If the resistance of iron be to that of copper as 825 : 112, compare the amounts of heat developed in a given time in these two portions of the circuit.

Answer. $\frac{\text{Heat in copper wire}}{\text{Heat in iron wire}} = \frac{121{\cdot}4}{1}$, nearly.

(9.) The current from a battery of 10 Grove's cells in series is sent through a piece of platinum wire whose resistance is ·058 ohm. The electromotive force of each cell is 1·9 volt, and the resistance of the whole circuit is 6·058 ohms. Find the amount of heat developed in the platinum wire in 5 minutes.

$$\text{In this case } C = \frac{19}{6{\cdot}058} \text{ ampères,}$$

$$\text{and } \therefore H = \frac{C^2 R t}{J} = \left(\frac{1{\cdot}9}{6{\cdot}058}\right)^2 \times \frac{{\cdot}058 \times 10^9 \times 300}{4{\cdot}2 \times 10^7}$$

$$= 40{\cdot}75 \text{ w.g.d. C}^\circ.$$

(10.) A piece of silver wire of exactly the same dimensions as the platinum wire was then substituted for it. If the resistance of the silver wire was ·0097 ohm, find the amount of heat developed in it by the above battery in 5 minutes. *Answer.* 6·93 w.g.d. C.°.

(11.) A Daniell's cell of 2 ohms' resistance had its poles connected by a piece of platinum wire of 4·7 ohms, and then by a piece of silver wire of the same dimensions, but of ·85 ohm resistance. The electromotive force of the cell being 1·12 volt, calculate the amount of heat developed in 5 minutes in each of these wires.

Answer. Heat in platinum = 9·381 w.g.d. C.°.
Heat in silver = 9·377 „ „

(12.) If the cell had been a very large one whose resistance was only ½ an ohm, what would have been the quantities of heat developed in the same time in each of these wires?

Answer. Heat in platinum = 1·557 w.g.d. C.°.
Heat in silver = 41·789 „ „

(13.) If a very small Daniell's cell had been used, whose electromotive force was the same but its resistance 8 ohms, what would have been the amount of heat developed in each of the above wires in 5 minutes?

Answer. Heat in platinum = 2·611 w.g.d. C.°.
Heat in silver = ·972 „ „

N.B. The student should compare the results of the last three examples, and he will notice that when the resistance of the cell is small, the silver wire becomes more heated than the platinum one, but the reverse is the case when the resistance of the cell is large. He should also notice that when the resistance of the cell is a mean proportional between the resistances of each of the wires, the heat developed in each of them is the same.

(14.) Twelve Grove's cells, each of which has an electromotive force of 1·9 volt and a resistance of ·28 ohm, are to be coupled up so as to develop the greatest possible amount of heat in a copper wire of ·21 ohm resistance. How must this be done?

Let C = current through extrapolar circuit,
R = resistance of " "
B = " " battery,
E = electromotive force of battery,
W = work done per second in the extrapolar circuit;

$$\text{then by Ohm's law } C = \frac{E}{B + R},$$

$$\text{and by Joule's law, } W = C^2 R = \frac{R E^2}{(B + R)^2}$$

and by a simple application of the Differential Calculus we find that, for W to be a maximum when R is variable, we must have B = R, or the battery must be so arranged that its resistance is equal to that of the extrapolar circuit.

Let x = number of files in the battery when arranged for this object; then $\frac{12}{x}$ = number of cells in each file, and $\frac{12}{x^2} \times {\cdot}28$ = resistance of battery;

$$\therefore {\cdot}28 \times \frac{12}{x^2} = {\cdot}21 \, ; \ \therefore x = 4,$$

and the battery must be arranged in 4 files of 3 each.

(15.) A wire of 6 ohms' resistance is to be heated by a battery of 6 Daniell's cells, the resistance of each of which is 4 ohms. How should they be arranged so as to develop the maximum amount of heat in a given time in this wire?

Answer. 2 files of 3 each.

(16.) The current from a battery of 10 Daniell's cells in series was sent through an extrapolar wire of a certain resistance. The resistance of each cell being 6 ohms, what must have been that of the wire so that the heat developed in it in a given time might be a maximum?

Answer. 60 ohms.

(17.) A current of ·75 ampère was sent for 5 minutes through a column of mercury whose resistance was ·47 ohm. The mass of the mercury was 20·25 grammes. The specific heat of mercury being ·03332, find the rise of temperature.

$$\text{Heat developed} = (\cdot 75)^2 \times \cdot 47 \times 300 = 79\cdot 313 \text{ joules}$$

$$= \frac{79\cdot 313}{4\cdot 2} \text{w.g.d. C.}^\circ;$$

and if θ = rise of temperature, m = mass in grammes, and s = specific heat of the substance, then

$$\text{H} = m\,s\,\theta = 20\cdot 25 \times \cdot 03332 \times \theta = \text{w.g.d. C.}^\circ;$$

$$\therefore \theta = \frac{79\cdot 313}{4.2 \times 20\cdot 25 \times \cdot 03332} = 27\cdot 987$$

$$= 28^\circ \text{ C., nearly.}$$

(18.) A coil of fine wire whose resistance was 5·23 ohms was placed in a calorimeter containing 1000 grammes of water, and a current of 5 ampères was sent through the coil for the space of 10 minutes. Supposing that the whole of the heat was taken up by the water and that its temperature was 10° C. at the beginning of the experiment, what was it at the end? *Answer.* 10°·19 C., nearly.

(19.) Supposing that the coil had a resistance of 20 ohms and that the same current had been sent through it for the space of 5 minutes, what would have been the rise of temperature of the water? *Answer.* $35^\circ\frac{5}{7}$ C., nearly.

(20.) A coil of very fine wire which had a resistance of 46·64 ohms was placed in 1000 grammes of water at 0° C. The current from a battery of 50 Daniell's cells in series, each of which had an electromotive force of one volt and a resistance of 6 ohms, was sent through the coil for 10 minutes. Supposing all the heat to be imparted to the water, find its rise of temperature.

Answer. 0°·14 C., nearly.

N.B.—If a comparison of the result of Examples 19 and 20 should at all perplex the student, he should notice that while in Example 19 the current strength is 5 ampères, in Example 20 it is 0·145 ampère, and that the heat developed varies as C^2.

(21.) A certain Griscom electric motor has a current of 3·9 ampères flowing through it, and the difference of potential between its terminals is 7·9 volts. Calculate the horse-power expended in it.

Work expended per second

$$= \text{E} \times \text{C} = \cdot 39 \times 7\cdot 9 \times 10^8 \text{ ergs}$$

$$= \frac{\cdot 39 \times 7\cdot 9 \times 10^8}{7\cdot 46 \times 10^9} = \frac{3\cdot 9 \times 7\cdot 9}{746}$$

$$= \cdot 0413 \text{ H.P.}$$

(22.) In another motor the difference of potential between the terminals was 8·9 volts and the current was 16·8 ampères. What horse-power was being absorbed by it?

Answer. ·2 H.P., nearly.

(23.) The difference of potential between the terminals of a certain electric motor is 26·4 volts and the current flowing through it is 8·1 ampères. Express in 'watts' the power absorbed by this motor.

'Watt' is the name given to the electromagnetic unit of power by the late Sir W. Siemens. It represents the amount of power conveyed by a current of one ampère through a difference of potential of one volt. We have then

$$\left.\begin{array}{ll} \text{One horse-power} & = 746 \times 10^7 \text{ ergs} \\ \text{One watt} & = 10^{-1} \times 10^8 = 10^7 \text{ ergs} \end{array}\right\} ;$$

$$\therefore \text{One watt} = \frac{\text{one H.P.}}{746}.$$

In the present case

$$\text{Power consumed in the motor} = 26\cdot 4 \times 8\cdot 1$$
$$= 213\cdot 84 \text{ watts.}$$

(24.) Calculate the power in watts which is being absorbed by a motor through which a current of 6·3 ampères is flowing with a difference of potential between its terminals of 10·6 volts. *Answer.* 66·78 watts.

(25.) A battery of 10 voltaic cells in series, each of which has an electromotive force of 1·12 volt and an internal resistance of 4 ohms, has its poles connected (1) by a wire of 40 ohms' resistance, and (2) by two wires of 80 ohms, each arranged in a multiple arc. Calculate the amount of heat developed in the wires per second in each case.

CASE I.—By Ohm's law $c = \frac{11·2}{80} = ·14$ ampère;

∴ heat developed in the wire per second
$= 40 \times (·14)^2 = ·784$ joule.

CASE II.—Current in battery $= ·14$ ampère;
∴ current in each wire $= ·07$ ampère;
∴ heat developed per second in each wire
$= 80 \times (·07)^2 = ·392$ joule.

(26.) What is the amount of heat developed in the battery per second in each of the above cases?

Answer. ·784 joule.

(27.) If a third wire of 80 ohms' resistance had been added in multiple arc, what would have been the amount of heat developed per second in the battery and in each of the wires?

Answer. Heat in battery = 1·129 joule; heat in each wire = ·251 joule.

(28.) A battery consisting of a series of 3 cells, each of which has a resistance of 2 ohms and an electromotive force of 1·04 volt, has its poles connected first by a short thick wire and then by a wire of 100 ohms' resistance. Find the amount of heat developed per second in the battery in each case. *Answer.* 1·6224 joule and ·0052 joule, nearly.

(29.) If a second wire of 100 ohms' had been joined to the poles of the battery in multiple arc, what would have been the amount of heat developed per second in each cell of the battery? *Answer.* ·00621 joule, nearly.

(30.) The resistance of the armature of a certain magneto-electric machine is 1·3 ohm, and the poles are connected by two branch circuits whose resistances are respectively 10 ohms and 12 ohms. Calculate the work expended per second in the armature and the work done in each of the external branches when a current of 26 ampères is flowing through the armature.

$$\left.\begin{aligned}\text{Strength of current in the 10 ohm branch} &= \frac{12}{22}\times 26\\ &= 14{\cdot}18 \text{ ampères}\\ \text{Strength of current in the 12 ohm branch} &= \frac{10}{22}\times 26\\ &= 11{\cdot}82 \text{ ampères}\end{aligned}\right\};$$

and since one-horse power is equivalent to $7{\cdot}46 \times 10^9$ ergs per second, ∴

$$\begin{aligned}\text{horse-power expended in 10 ohm branch} &= \frac{10\times(14{\cdot}18)^2}{746}\\ &= 2{\cdot}7 \text{ H.P.}\\ \text{horse-power expended in 12 ohm branch} &= \frac{12\times(11{\cdot}82)^2}{746}\\ &= 2{\cdot}25 \text{ H.P.}\\ \text{horse-power expended in heating armature} &= \frac{1{\cdot}3\times(26)^2}{746}\\ &= 1{\cdot}178 \text{ H.P.}\end{aligned}$$

(31.) If the resistance of each of the exterior branches were made equal to 2·6 ohms, and the same current as before were made to circulate in the armature, calculate the horse-power expended in the armature and in each branch of the exterior circuit.

Answer. Horse-power in each branch = ·589 ;
horse-power in armature = 1·178.

(32.) The resistance of a certain voltaic battery is 14 ohms, and its electro-motive force is 100 volts. Its poles are first short-circuited by a thick wire, and are next connected by a circuit of 12 ohms' resistance. Calculate the quantity of heat developed per second in the battery in each case. *Answer.* 714·3 joules and 207·1 joules.

(33.) The poles of a certain dynamo machine whose internal resistance is ·13 ohm are connected in multiple arc by 20 branch circuits, each of 100 ohms' resistance, and in each of which a current of one ampère is maintained by the dynamo. Calculate the work done per second in the machine and in each circuit.

Answer. 52 watts and 100 watts.

(34.) The resistance of an electric motor is 3·2 ohms and the current flowing through it is 11·2 ampères. The horse-power given out by it is ·116 H.P. What is its *efficiency* per cent.?

The *efficiency* of the motor is the ratio of the amount of power given out by it to the amount of power expended in the motor in driving it.

In the present case

$$\text{Work done in motor per second} = \frac{\text{R C}^2}{746} = \frac{3{\cdot}2 \times (11{\cdot}2)^2}{746}$$

$$= {\cdot}538 \text{ H.P.};$$

$$\therefore \text{ efficiency per cent.} = \frac{11{\cdot}6}{{\cdot}538} = 21{\cdot}6, \text{ nearly.}$$

(35.) Calculate the percentage efficiency of an electric motor which is giving out ·115 H.P. when a current of 10·1 ampères is flowing through it, its resistance, when warm, being 4·2 ohms. *Answer.* 20·024, nearly.

(36.) A copper ring whose mean diameter is 25 centimetres and whose cross section is one square centimetre is made to revolve 100 times in a minute between the poles of an electromagnet in a field of strength 500. Taking the specific resistance per cubic centimetre of copper as 1·616 microhms, calculate the power required to drive the ring.

The resistance of the ring

$$= \text{R} = 25\,\pi \times 1{\cdot}616 \text{ microhms} = 1{\cdot}269 \times 10^{-4} \text{ ohm.}$$

The average electromotive force per second

$$= 4 n \pi r^2 I = 4 \times \frac{1000}{60} \pi \times \left(\frac{25}{2}\right)^2 \times 500$$

$$= 1{\cdot}636 \times 10^7 \text{ C.G.S. units} = {\cdot}1636 \text{ volt;}$$

$\therefore$ average current in ring is

$$C = \frac{E}{R} = \frac{{\cdot}1636}{1{\cdot}269 \times 10^{-4}} = 1289 \text{ ampères.}$$

The power that must be expended to drive the ring against the electrical forces over and above the friction is

$$\text{Power} = \frac{R C^2}{746} = \frac{1{\cdot}269 \times 10^{-4} \times (1289)^2}{746}$$

$$= {\cdot}283 \text{ H.P., nearly.}$$

(37.) [illegible] amount of heat would be generated in this ring per second? *Answer.* 50·2 w.g.d. C.°, nearly.

(38.) A coil of 200 turns of copper wire whose mean radius was 10 centimetres and its resistance 5 ohms was mounted between the poles of an electromagnet in a magnetic field of strength 800. What power would be required to drive it round at the rate of 180 turns a minute?

Answer. $9{\cdot}754 \times 10^{-3}$ H.P., nearly.

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FIG. 2.

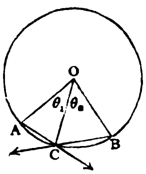


FIG. 7.

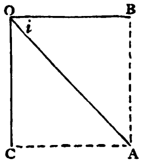
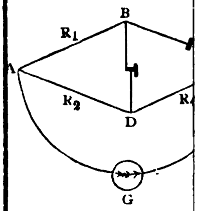


FIG. 12.



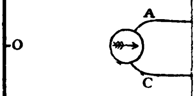


FIG. 5.

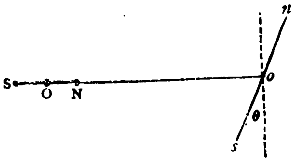


FIG. 10.

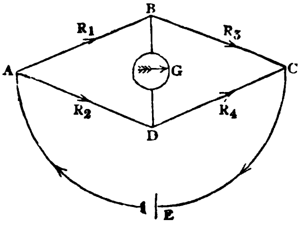


FIG. 15.

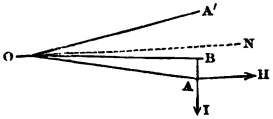
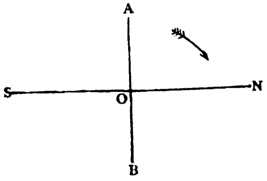


FIG. 20.



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